B. M. MA
APRIL 1972

AFOSR - TR - 72 - 0962

[redacted]

ISU – ERI – AMES – 72075

VARIOUS PLASMA-STABILIZATION METHODS FOR CONTROLLED THERMONUCLEAR FUSION: COMPARISON AND ANALYSIS

Project 895-S

79

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
IOWA STATE UNIVERSITY ENGINEERING RESEARCH INSTITUTE AMES, IOWA 50010	UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

VARIOUS PLASMA-STABILIZATION METHODS FOR CONTROLLED THERMONUCLEAR FUSION: COMPARISON AND ANALYSIS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Scientific Interim

5. AUTHOR(S) (First name, middle initial, last name)

B M MA

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
April 1972	74	70

8a. CONTRACT OR GRANT NO	9a. ORIGINATOR'S REPORT NUMBER(S)
AFOSR-71-2072	
b. PROJECT NO. 9752-01	ISU-ERI-AMES-72075
c. 61102F	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d. 681308	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
TECH, OTHER	AF Office of Scientific Research (NAM) 1400 Wilson Boulevard Arlington, Virginia 22209

13. ABSTRACT

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UNCLASSIFIED

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
CONTROLLED THERMONUCLEAR FUSION						
FUSION ENERGY						
PLASMA STABILIZATION						
MAGNETIC SHEAR STABILIZATION						
MAGNETIC WELL STABILIZATION						
DYNAMIC STABILIZATION						
FEEDBACK STABILIZATION						

UNCLASSIFIED

Security Classification

ENGINEERING
RESEARCH

ENGINEERING
RESEARCH
ENGINEERING
RESEARCH
ENGINEERING
RESEARCH
ENGINEERING
RESEARCH

[redacted]

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April 1972

Submitted to:
the Air Force Office of Scientific Research
AFOSR-71-2072

ISU-ERI-AMES-72075
ERI Project 895-S

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IOWA STATE UNIVERSITY AMES

ABSTRACT

To improve the stability and containment time of a plasma in a magnetic field, the plasma stabilization for controlled thermonuclear fusion must be developed. The methods or principles of magnetic shear, magnetic well, dynamic, and feedback stabilization are of primary importance and will be given primary consideration. The objectives of this research are to:

- review various stabilizing methods,
- compare the effectiveness, advantages and disadvantages of some stabilizing methods, and
- deduce from the results obtained a new idea or original method for understanding plasma instability and improving plasma stabilization.

1. INTRODUCTION

Plasma stabilization during developing controlled fusion is one of the most difficult achievements in thermonuclear research. To improve the stability and containment time of a plasma contained in a magnetic field, various stabilizing methods have been and are being developed. Among them, of primary importance, are magnetic field configuration stabilization, dynamic stabilization and feedback stabilization. For magnetic field configuration stabilization, the plasma may be made self-stabilizing in some complex magnetic fields. For dynamic stabilization, the high-frequency (or radiofrequency) field interacts with the entire plasma to establish a new dynamically stable equilibrium. For feedback stabilization, the plasma equilibrium remains the same and the time-varying suppression signal interacts only with the unstable part of the plasma. The reality of developing a controlled fusion reactor depends mainly on the development of an appropriate magnetic field configuration and the availability of remote control and detection methods of the dynamic stabilization or feedback stabilization for a thermonuclear plasma.

There are many methods proposed for the magnetic field configuration stabilization, the dynamic stabilization and the feedback stabilization of a thermonuclear plasma. Only some of the main methods of plasma stabilization will be discussed, analyzed and compared in this research work.

2. MAGNETIC FIELD CONFIGURATION STABILIZATION

Plasma stabilization by magnetic field configuration consists mainly of the following methods: (a) internal axial magnetic field; (b) rotational transform devices; (c) screw pinch; (d) magnetic shear; (e) magnetic well-minimum B (open-end system), magnetic well-minimum average B (closed-end system); and (f) multipole configuration.

2.1 Internal Axial Magnetic Field

To improve the stability of a linear or toroidal pinched plasma, an internal axial magnetic field was early introduced.[1,2] Experiments show that the introduction of an axial magnetic field in plasma can delay instability of the pinched discharge. Certain ratios of the axial and azimuthal magnetic fields, B_z/B_θ, and some magnitude of the pinch ratio, r_o/r, (less than 5) must also be met, where B_z and B_θ are the axial and azimuthal magnetic fields and r_o and r are the radii of the discharge tube and the pinched plasma, respectively.

Interdiffusion (or intermixing) of axial and azimuthal magnetic fields must be taken into consideration when B_z and B_θ diffuse into each other in the presence of a finite plasma resistivity, η. The interdiffusion of the magnetic fields has two important effects: (a) The redistribution of magnetic field is in a transient state. As a result, the pinched plasma tends to become unstable, due to continuous decrease in B_z. (b) The magnetic field diffusion energy loss due to resistivity (η) in the current sheath tends to heat the plasma, i.e., the ohmic heating.

Consider a cylindrical plasma sheath of unit length and radius r, contained by the axial and azimuthal magnetic fields in a long discharge tube. To evaluate the field time constant, τ, for diffusion of the axial field flux out of the cylindrical sheath, the energy, W, of magnetic field B (in MKS units) is given by

$$W = \pi r^2 \frac{B^2}{2\mu_o} \tag{2.1}$$

where μ_o is the permeability in vacuum. By differentiating B with respect to time, t, the rate of the energy loss is expressed as

$$-\frac{\partial W}{\partial t} = -\frac{\pi r^2}{2\mu_o}\frac{\partial B^2}{\partial t} \quad . \tag{2.2}$$

This relation is equal to resistive power, P, in the current sheath to heat the plasma

$$P = I^2 R = I^2 \eta \frac{2\pi r}{\delta} \tag{2.3}$$

where I and R are the electrical current and resistance of the plasma and δ is the sheath thickness. Assume that the rate of magnetic energy loss by diffusion is entirely converted into the resistive power of ohmic heating in the plasma, from Eqs. (2.2) and (2.3), it follows that

$$I^2 = -\frac{r\delta}{4\eta\mu_o}\frac{\partial B^2}{\partial t} \tag{2.4}$$

A plasma sheath can be considered as a single-turn solenoid, according to the Biot-Savart law that current and magnetic field are related by

$$B = \mu_o I \tag{2.5}$$

Introduction of Eq. (2.5) into Eq. (2.4) gives

$$\frac{1}{B^2}\frac{\partial B^2}{\partial t} = -\frac{\mu_o r\delta}{4\eta} .$$

Integrating this and using the initial condition $B = B_o$ at $t = 0$, the time constant τ_o and the field time constant τ related to B^2 and B are, respectively

$$B^2 = B_o^2\, e^{-t/\tau_o} , \quad B = B_o\, e^{-t/\tau} \tag{2.6}$$

where

$$\tau_o \equiv \frac{\mu_o r\delta}{4\eta} , \qquad \tau = \frac{\mu_o r\delta}{2\eta}$$

so that $\tau = 2\tau_o$.

It is obvious that the diffusion of the confining magnetic field (or the drift of charged particles) from the plasma can be slowed down if η is small and r is large. This result can be generalized for various devices: (a) it will take a longer time for B_z to diffuse in a large discharge tube than in a small one, and (b) the containment of a stable discharged plasma requires rapid initial heating to attain a high temperature for small η.

2.2 Rotational Transform Devices

It is well known that the rotational transform was first introduced in the stellarator machine.[3] The stellarator can produce a rotational transform because of its "figure 8" configuration. In the presence of an axial magnetic field with a rotational transform, the electrostatic fields and charge separation due to the oppositely directed drifts of positive ions and negative electrons in the stellarator system can be considerably

suppressed. Magnetic transform and particle transform can also serve the purpose of the rotational transform.[4]

To compensate for the ion and electron drifts in the curved region of a race track type of toroidal device, a corrugated magnetic field can also produce the effect of a rotational transform. The corrugated magnetic fields can be realized by a suitable spacing of the winding of a number of solenoidal coils which produce the axial magnetic field when energized.

2.3 Screw Pinch

Screw pinch is a dynamic pinch wherein an external axial magnetic field, B_z, is trapped outside the plasma. This axial magnetic field and the azimuthal magnetic field, B_θ, combine to form a helical screw-type magnetic field to confine the plasma.

Early theoretical studies indicated that the sausage instability ($m = 0$) of a constricted pinch discharge and the kink instability ($m = 1$) of short wavelength could be stabilized by an applied external axial magnetic field as an alternative to the internal axial magnetic field. The possibility of stabilizing a pinched discharge by an external axial magnetic field led to the development of the screw dynamic pinch.[5-7]

The partially stabilized screw pinch can be realized by the application of the external axial field simultaneously with the azimuthal field of the discharge. This is achieved in a linear or toroidal discharge tube of the usual type, modified by imparting a slight helical pinch to the return current outside of the tube. The conductor consists of a number of copper-braided strips (as in the Tokamak TM-3) insulated

from one another, arranged along the length of the tube with a slant $rd\theta/dz$ equal to 0.1-0.2 (i.e., along the z-axis at radius r and angle θ). As a result, an external magnetic field is generated by the discharged current, and the total magnetic field (both B_θ and B_z) contains and heats the plasma in a helical fashion.

From the facts that the region outside the constricted plasma is a partial vacuum and has appreciable conductivity, the concept of screw dynamic pinch evolved. To simplify analysis, a pressureless ionized gas of infinite conductivity outside the pinch column proper is assumed. The radial equilibrium equation of magnetic pressure for a screw dynamic pinch distribution in the plasma column is given by

$$B_z \frac{dB_z}{dx} + \frac{b_\theta}{x} \frac{d(xB_\theta)}{dx} = 0 \tag{2.7}$$

where $x = r/r_o$ and r_o = radius of the linear pinch tube.

Given the properties of adiabatic invariant, the magnetic fluxes, ϕ_θ and ϕ_z, between a given point and the pinch axis in the plasma must be conserved. Hence, ϕ_θ has a definite functional dependence on ϕ_z. Then, from the orthogonality principle, the azimuthal and axial magnetic fields can be expressed in terms of ϕ_θ and ϕ_z (similar to the properties of the stream function and the potential function in fluid mechanics)

$$B_\theta = \partial\phi_\theta/r_o\partial x \ , \quad B_z = \partial\phi_z/r_o^2 x\partial x \tag{2.8}$$

which satisfy Eq. (2.7) of the pinched plasma.

Since

$$\frac{\partial\phi_\theta}{r_o\partial x} = \frac{d\phi_\theta}{d\phi_z} \frac{\partial\phi_z}{r_o\partial x} \ . \tag{2.9}$$

Introduction of Eqs. (2.8) into Eq. (2.9) yields

$$B_\theta = \nu r_o x B_z \tag{2.10}$$

where $\nu = d\phi_\theta/d\phi_z$ which is a constant at any point in the plasma. This equation represents the relationship between B_θ and B_z at any point x in the plasma. In a linear screw pinch, the return conductor of the discharged currents is a helix outside the plasma tube. If the radius of the pinch tube is taken as unity, then the wall condition of the screw pinch becomes

$$B_\theta = \nu_o B_z \tag{2.11}$$

where

$r_o x = 1$, and

$\nu_o = (d\phi_\theta/d\phi_z)_{x=1}$ = constant.

Substitution of Eq. (2.10) into Eq. (2.7), and rearrangement, give the relations

$$B_\theta = b\ \nu\ r_o x B_{zo},\ B_z = b\ B_{zo} \tag{2.12}$$

where

$b = (1 + \nu_o^2\ r_o^2)/(1 + \nu_o^2\ r_o^2 x^2)$,

at $x = 1$, $b = 1$, and

$B_z = B_{zo}$

on the wall of the test tube. For given values of r_o and ν, the radial distribution of B_θ/B_{zo} and B_z/B_{zo} at any point x of the screw pinch can be determined.

In the above analysis, the conductivity outside the pinch column has been assumed as infinity. Actually, a finite resistivity is present. Consequently, the interdiffusion between B_θ and B_{zo} as well as the decay of B_{zo} will occur as discussed in the method of internal magnetic field stabilization.

A theoretical study indicated that a screw or diffuse pinch with axisymmetric configuration operating above the Kruskal-Shafranov current limit can be stable if $\beta > 0.1$ where $\beta = 2\mu_o p/(B_z^2 + B_\theta^2)$ and p = plasma kinetic pressure. For the purpose of comparison, some computed results of a numerical example for the stabilized pinch with external current and the screw pinch with external current are shown in Fig. 1. The pinched plasma in both cases of the diffuse pinch is stable. The stability is possible for case (a) where β is in the range 0.05 to 0.10, while for case (b), β must be greater than 0.1. The magnitudes of the selected magnetic fields B_z and B_θ are typical values and the pitch is defined as $P = rB_z/B_\theta$.

Most of the present laboratory devices with high β toroidal plasma containment exhibit the flute or interchange instability (m = 1). Dynamic stabilization of the m = 1 instability for high β screw pinch was studied experimentally.[8] In the experimental work, two basic dynamic stabilization methods were performed: (a) The oscillating, axially uniform high-frequency component of the B_z field, and (b) the oscillating axial component current methods. For case (a), special attention is given to the possibility of working in resonance with natural plasma oscillation (having relatively low frequency) to save energy input required by a high frequency oscillator. For case (b) the effects of dynamic shear on the flute instability is particularly considered. These methods are similar to that of dynamic stabilization to be discussed later.

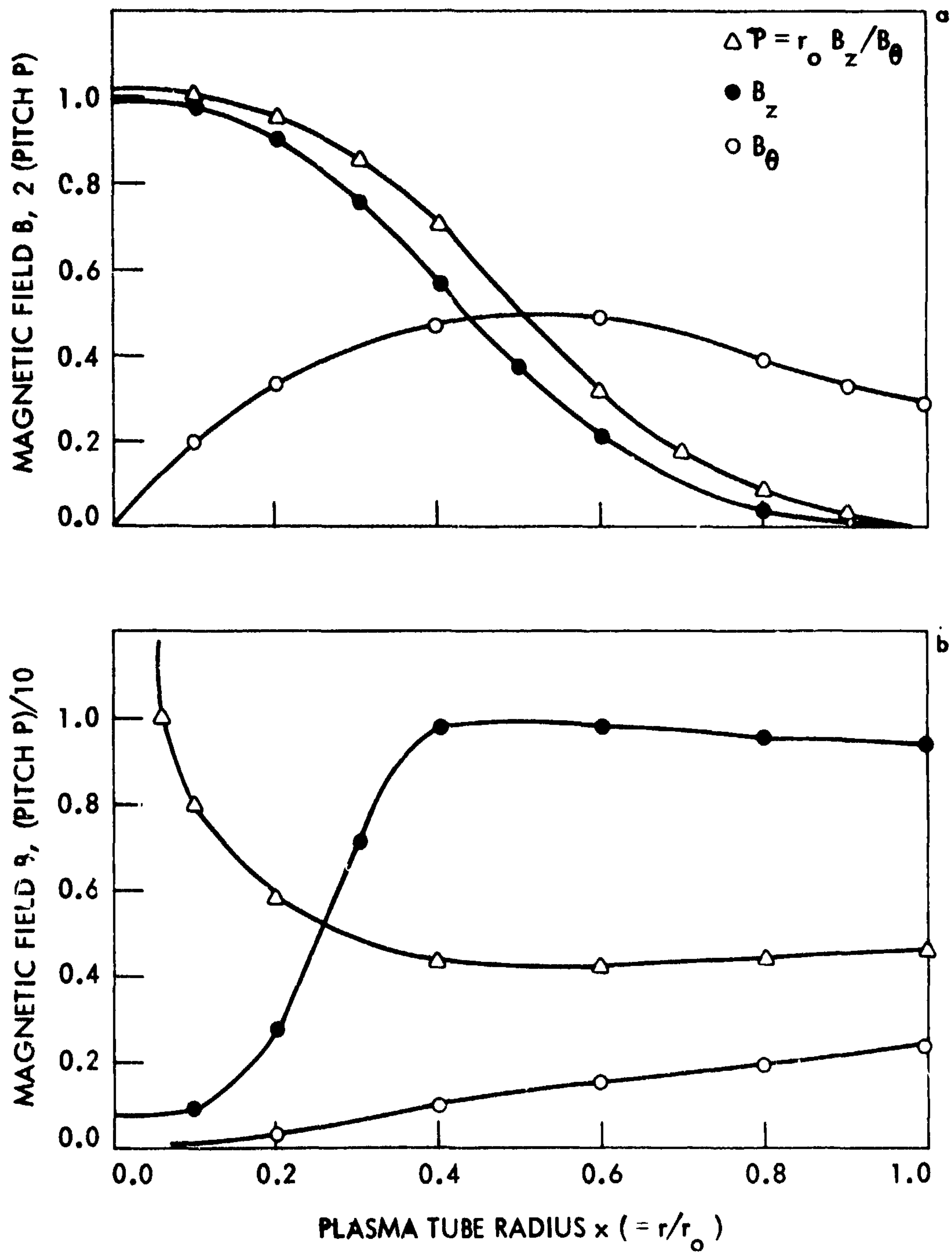

Fig. 1. Typical magnetic field and magnetic pitch of the stabilized diffuse pinches vs plasma tube radius: (a) stabilized pinch with external current; (b) screw pinch with external current.

In the oscillating axial magnetic field stabilization method, the oscillating component of the axial magnetic field $\tilde{B}_{zo} \sin \omega_{st} t$ can be produced and superimposed on the base axial magnetic field B_{zo} in a common coil to obtain the total axial magnetic field, $B_z = B_{zo} + \tilde{B}_{zo} \sin \omega_{st} t$ where $\tilde{B}_{zo}$ is the amplitude of the oscillating axial field, ω_{st} is the stabilizing frequency and t is the time. The ratio $\tilde{B}_{zo}/B_{zo}$ can be varied between 0.05 and 0.20. The range of ω_{st} can be adjusted from 1-1.7 MHz. The growth rate of the flute instability is moderately suppressed after the oscillating axial magnetic field has been applied. In addition, the high frequency energy input could be saved and the plasma column breakdown by wall contact could be avoided if the resonance condition between the oscillated plasma motion and the natural radial plasma oscillation could, in principle, be achieved. In the oscillating axial current, the total axial current, $I_z = I_{zo} + \tilde{I}_{zo} \sin \omega_{st} t$, is fed into the screw pinch using the same electrodes for the high frequency oscillating $\tilde{I}_{zo}$ and the quasi-steady current I_{zo}. $\tilde{I}_{zo}$ and I_{zo} have comparable amplitudes from 15-50 kA. The stabilization frequency ω_{st} is about 1 MHz. A reduction of the m = 1 growth rate is observed after $\tilde{I}_{zo}$ is superimposed on the screw pinch.

The growth rate or instability frequency, $f_{in} = \omega_{in}/2\pi$, for the m = 1 mode instability in a linear (circles) and a toroidal (dots, aspect ratio = 5) screw pinch as a function of total axial current I_z is shown in Fig. 2. The growth rate and the wavelength of the m = 1 instability are very similar in a linear and a toroidal high β screw pinch for adequate, corresponding plasma parameters. Neither the end effects nor the effects of toroidal curvature strongly influence the m = 1 growth rate. The

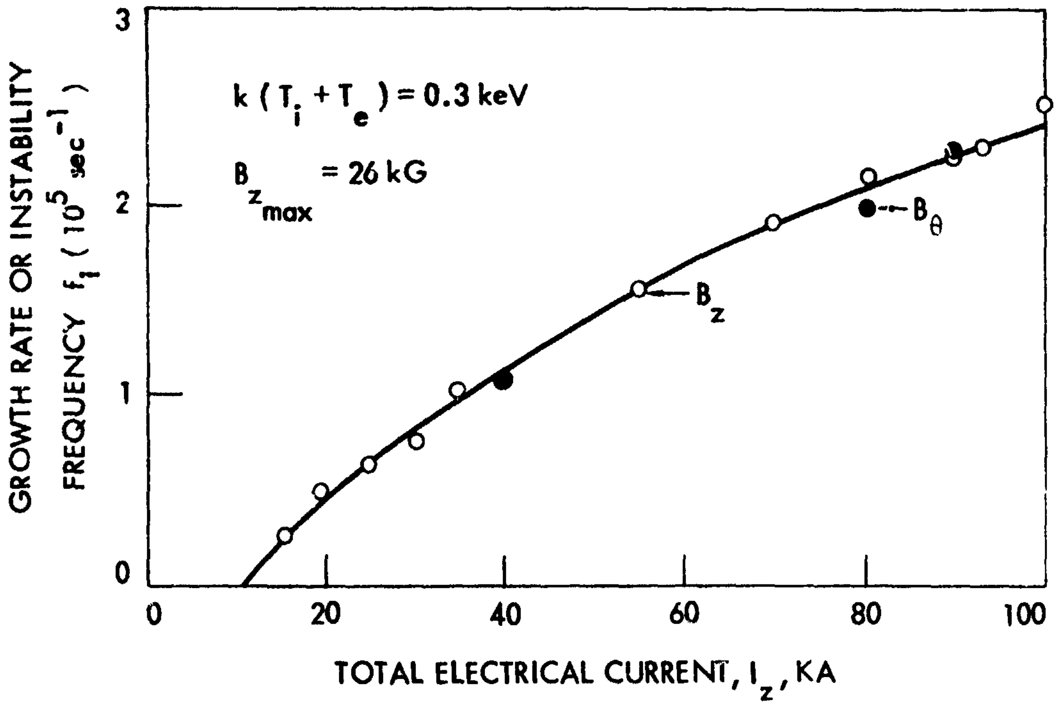

Fig. 2. Growth rate of the m = 1 instability in a linear and toroidal screw pinch vs total electric current.

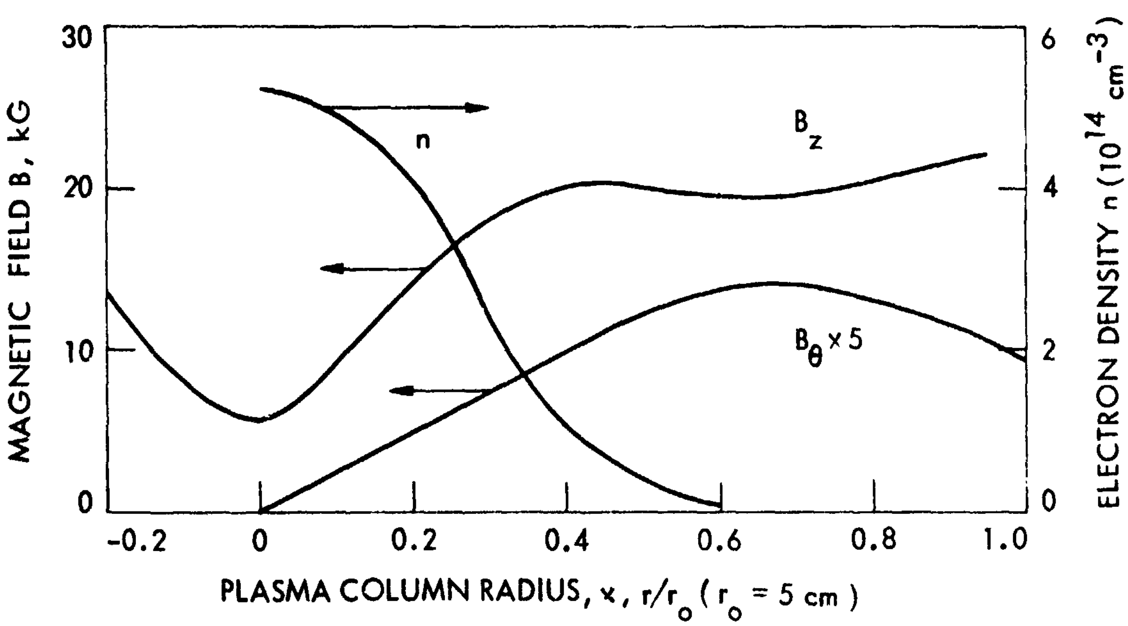

Fig. 3. Typical radial distributions of magnetic fields and electron density in a linear screw pinch.

observed radial distributions of the magnetic fields B_θ, B_z and the plasma density n in a linear screw pinch at t = 2 μsec, p = $45(10^{-3})$ Torr and T_i = 0.20 keV is given in Fig. 3. It is seen that the dynamic effect of screw pinch can heat the ions of a plasma at very high rates, and the dynamic shear tends to stabilize the plasma. The above observed data are in good agreement with the calculated results from the theoretical analysis.[9,10]

2.4 Magnetic Shear

Magnetic shear is appreciably effective in controlling the gross, universal, or drift instability of a thermonuclear plasma. It is the electrical short-circuit principle whereby any separation of charges from positive ions and negative electrons can be neutralized by short-circuiting the charges. As a result, the plasma can be appreciably stabilized.[11,12]

The magnetic shear is produced by the twist of magnetic field lines in the plasma. The degree of twist of the magnetic lines can be made to vary with distance from the central axis, the farther from the axis, the deeper the pitch of the lines, and vice versa.

The magnetic shear of the magnetic field configuration stabilization can be produced by a number of methods. The two most common methods are the reversed magnetic field device and the helical magnetic windings. The influence of a magnetic shear tends to make the radial pressure gradient positive so that Suydam's necessary condition of plasma stability may be satisfied.[13,14] In most cases, the magnetic shear is used to stabilize the gross, universal or drift instability.[15,16] The gross or universal instability (independent of plasma configuration) due to temperature and density gradients is synonymous with confinement, and the drift

instability is the diamagnetic drift caused by the gradients.

The drift instability arises from the Landau damping mechanism involving particles whose velocities along the magnetic field resonate with wave velocity. Sometimes, this damping mechanism also involves energy transfer from the resonant particles to the oscillating waves. The drift instability can occur under certain conditions: (a) a large density gradient exists in the plasma; (b) a steep transverse particle temperature gradient exists; (c) the temperature gradient opposes the density gradient of the plasma; or (d) a sufficient, local electric field exists or electron current flows along the magnetic field.

In the introduction of the wave packet model, a local instability or perturbation of the plasma is assumed to be composed of wave packets $\Phi(\vec{r}, t)$ of the form

$$\Phi(\vec{r}, t) = \phi(\vec{r}) \exp i(\vec{k}\cdot\vec{r} - \omega t) \tag{2.13}$$

where

$\Phi(\vec{r})$ = the spatial perturbation as a function of the space vector $\vec{r}$,

$\vec{k}$ = the spatial wave number,

ω = angular frequency of the plasma, and

t = time.

A general stability criterion of the wave packets is that the wave energy of the wave packets should not grow or exponentiate many times as they move through some region of local instability.

To stabilize the drift instability, the sheared magnetic field in relation to the wave packet has two main effects: (a) an effective increase in the ion Landau damping available for plasma stabilization is through the propagation of the wave packet, or the expansion of the potential well. The

potential well can expand into the region where the Landau damping by the interaction with resonant ions is strong; and (b) the decrease in wave energy of the wave packet, or the shrinking of the wave packet as it propagates through some local region and gradually vanishes.

In stability analysis, a plasma with a density gradient dn/dx (n = plasma density) in the x direction, an unperturbed magnetic field vector $\vec{B} = B_o\left[\vec{e}_z + (x/L_s)\vec{e}_y\right]$ and a Maxwellian velocity distribution is considered, where $B_0\vec{e}_z = \vec{B}_z$ = main axial magnetic field, $B_0(x/L_s)\vec{e}_y$ = shearing magnetic field, B_0 = the scalar magnetic field, $\vec{e}_y$ and $\vec{e}_z$ are the unit vectors in the y and z directions, and L_s is the characteristic shear or shearing distance.

With the Boltzmann distribution function $f_j(\vec{r}, \vec{v}, t) = f_j$, the Vlasov equation is given (in Mks units) by

$$\frac{\partial f_j}{\partial t} + \vec{v}\cdot\nabla f_j + \frac{e_j}{m_j}(\vec{E} + \vec{v} \times \vec{B})\nabla_v f_j = 0 \ , \qquad (2.14)$$

where $\vec{v}$ = velocity vector in space, e = unit electric charge, m = particle mass, and $\vec{E}$ = self-consistent electric field strength. The vector $\vec{E}$ satisfies the Poisson equation of charge (or the Gaussian law)

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \sum_j e_j n_j$$

in which ϵ_0 = permitivity in a vacuum and n_j is evaluated from the $j\underline{th}$ species particle distribution function f_j:

$$n_j = \int f_j(\vec{r}, \vec{v} \ \ t)d^3v.$$

If there is no initial equilibrium electric field, $\vec{E}_0 = 0$, the linearization of Eq. (2.14) yields the first-order equation for f_{1j} (the zero-order equation for f_{0j} is omitted):

$$\frac{\partial f_{1j}}{\partial t} + \vec{v} \cdot \nabla f_{1j} + \frac{e_j}{m_j} (\vec{v} \times B_0) \nabla_v f_{1j} = - \frac{e_j}{m_j} (E_1 + \vec{v} \times \vec{B}_1) \cdot \nabla_v f_{0j} \; . \tag{2.15}$$

By the method of characteristics, the solution of Eq. (2.15) can be written as

$$f_{1j} = - \frac{e_j}{m_j} \int (\vec{E}_1 + \vec{v} \times \vec{B}_1) \cdot \nabla_v f_{0j} \, dt, \tag{2.16}$$

where the integral is performed along the particle orbit in the lowest order. It is assumed that a time space dependence of the perturbation is

$$f_{1j} = (f_{1j})_0 \exp i(\vec{k} \cdot \vec{r} - \omega t) \tag{2.17}$$

in which $(f_{1j})_0$ is a constant. By combining the solution of the Vlasov equation and the well-known Maxwell equations, the dispersion relation for low-frequency modes of the plasma can be given by

$$(\omega - \omega_e)\left[\left(1 - i \frac{\epsilon_0^{-1/2} \omega}{2|k_{||}| v_e}\right) - \frac{T_e}{T_i} \left(1 - \frac{\omega_e}{\omega}\right) \frac{k_{||}^2 v_i^2}{2\omega^2} I_0(b) e^{-b}\right]$$
$$\left[\omega(\omega - \omega_e) - \frac{k^2}{k_\perp^2} \frac{b k_{||}^2 v_A^2}{1 - I_0(b) e^{-b}}\right] = \frac{k^2}{k_\perp^2} b k_{||}^2 v_A^2 \frac{T_e}{T_i} \left(1 - \frac{\omega_e}{\omega}\right), \tag{2.18}$$

where

$$\omega_e = \frac{k_y T_e}{eB} \frac{dn}{n dx} = \text{electron frequency},$$

$$v_e = \left(\frac{2T_e}{m_e}\right)^{\frac{1}{2}} = \text{electron thermal velocity},$$

$\omega_i^* = \frac{k_y T_i}{eB} \frac{dn}{ndx}$ = ion frequency,

$v_i = \left(\frac{2T_i}{m_i}\right)^{\frac{1}{2}}$ = ion thermal velocity,

T_e, T_i = electron and ion temperatures, or kinetic temperatures,

$k_\perp = (k_x^2 + k_y^2)^{\frac{1}{2}}$ = wave number perpendicular to $\vec{B}_z (= B_0\vec{e}_z)$,

$k_\| = k_z + \frac{x}{L_s} k_y$ = wave number for shear effects,

$b = \frac{1}{2}k^2a_i^2 \simeq \frac{1}{2}k_\perp^2 a_i^2$,

a_i = ion gyroradius or gyromagnetic radius,

v_A = the Alvén velocity,

$I_0(b)$ = zero order of the first kind of modified Bessel function,

k_x, k_y, k_z = wave numbers in the x, y and z directions.

In deriving the dispersion relation, the following basic assumptions have been made:

- the space-time dependence of perturbation varies as the behavior of the wave packet defined above,
- the characteristic length of density gradient, $r^{-1} = \left|\frac{1}{n}\frac{dn}{dx}\right| \ll k_x$. The amounts of magnetic shear required to stabilize drift instability due only to density gradient for the plasma $m_e/m_i < \beta \ll 1$ is sought.
- the electron gyromagnetic radius is much smaller than the wavelength.
- $k_\perp \gg k_z$, $v_i \ll |\omega/k_z| \ll v_e$
- $k\lambda_D \ll 1$ and $\omega \ll \Omega_i$, Ω_e where λ_D = the Debye length and $\Omega_i = \frac{eB}{m_i}$, $\Omega_e = \frac{eB}{m_e}$, the ion and electron cyclotron frequencies.

If $T_i = T_e$, $2v_i^2/v_A^2 = \beta$ (= kinetic pressure/magnetic pressure of the plasma), $k_\perp^2 \gg k_\parallel^2$, $k_\perp^2 \simeq k^2$, and $c^2k_\perp^2/k_\parallel^2 = bv_A^2$ (c = velocity of light), then the dispersion relation reduces to

$$2 - \left(1 - \frac{\omega_e}{\omega}\right)\left(1 + \frac{k_\parallel^2 v_i^2}{2\omega^2}\right) I_0(b)e^{-b} - i\,\frac{\epsilon_0^{-\frac{1}{2}}(\omega - \omega_e)}{2|k_\parallel|v_i}$$
$$- \frac{\beta(\omega - \omega_e)}{2bk_\parallel^2 v_i^2}\left(1 - \frac{\epsilon_0^{-\frac{1}{2}}\omega}{2|k_\parallel|v_e}\right)\left[1 - \left(1 - \frac{k_\parallel^2 v_A^2}{2\omega^2}\right) I_0(b)e^{-b}\right] = 0. \tag{2.19}$$

Based on the dispersion equations, some limiting cases for long and short wavelengths of plasma oscillation can be discussed.

2.4a Long Wavelength Oscillation

In this case, $b \ll 1$ and $I_0(b)e^{-b} \simeq 1 - b$, then for $\omega_i = \omega_e$, Eq. (2.19) becomes

$$\left(1 - i\,\frac{\epsilon_0^{-\frac{1}{2}}\omega}{2|k_\parallel|v_e} - \frac{k_\parallel^2 v_i^2}{2\omega^2}\right)\left[\omega(\omega - \omega_e) - k_\parallel^2 v_A^2\right] = bk_\parallel^2 v_A^2 . \tag{2.20}$$

This equation consists of three modes of plasma oscillation: (a) the electrostatic mode, (b) the slow Alfvén mode, and (c) the fast Alfvén mode. For the electrostatic mode, it is assumed that $\omega^2 \ll k_\parallel^2 v_A^2$ and $b \ll 1$. The last term in Eq. (2.20) is negligible compared with other terms. An obvious solution of the resulting expression is $\omega_1 = -\omega_e$ as ω_1 and ω_e propagate in opposite directions.

Introduction of $\omega_1 + \delta\omega_1$ for a perturbed frequency ω into Eq. (2.20) gives

$$\delta\omega_1 = -\omega_e\left(\frac{2b}{1 - 2\omega_e^2/k_\parallel^2 v_A^2} - \frac{k_\parallel^2 v_i^2}{\omega_e^2}\right)\left(1 - i\,\frac{\epsilon_0^{-\frac{1}{2}}\omega_e}{2|k_\parallel|v_e}\right). \tag{2.21}$$

Hence, the perturbation condition of the plasma (as $\omega_e \neq 0$) is

$$\frac{2b}{1 - 2\omega_e^2/k_{\|}^2 v_A^2} > \frac{k_{\|}^2 v_i^2}{\omega_e^2}, \text{ or}$$

$$2\omega_e^2 < k_{\|}^2 v_A^2 \text{ and } 2b \gtrsim k_{\|}^2 v_i^2/\omega_e^2$$

which shows that the unstable electrostatic modes propagate almost perpendicularly to the axial magnetic field and appreciable magnetic shear effect $k_{\|}$ is required to suppress the drift instability.

For the continuous (or transient) Alfvén modes, the second term on the left-hand side of Eq. (2.20) approaches zero, i.e., $\omega(\omega - \omega_e - k_{\|}^2 v_A^2 = 0$. The roots of this quadratic equation represent the approximate frequencies of the slow and fast Alfvén modes:

$$\omega_2, \omega_3 = \frac{\omega_e}{2} \pm \left[(\omega_e/2)^2 + k_{\|}^2 v_A^2\right]^{\frac{1}{2}} . \tag{2.22}$$

Introducing $\omega_2 + \delta\omega_2$ and $\omega_3 + \delta\omega_3$ for ω into Eq. (2.20), respectively, the values of $\delta\omega_2$ and $\delta\omega_3$ are

$$\delta\omega_2, \delta\omega_3 = \mp \frac{b k_{\|}^2 v_A^2}{\left(\omega_e^2/4 + k_{\|}^2 v_A^2\right)^{\frac{1}{2}}} \left[\frac{2}{3} + \frac{\omega_2 - \omega_e}{2\left(\omega_2^2 + \omega_e^2\right)}\left(1 + i\,\frac{\pi^{\frac{1}{2}}}{2}\,\omega_2 |k_{\|} v_e|\right)\right] . \tag{2.23}$$

The perturbation condition for Eqs. (2.22) require $\omega_e^2 > 4k_{\|}^2 v_A^2$. For a given value of v_A, a relatively small magnetic shear is needed to inhibit the drift instability. The respective frequencies of the electrostatic mode and the slow and fast Alfvén modes of plasma oscillation are represented by $\omega_1 + \delta\omega_1 (= -\omega_e + \delta\omega_1)$, $\omega_2 + \delta\omega_2$ and $\omega_3 + \delta\omega_3$.

2.4b Short Wavelength Oscillation

In the oscillation of short wavelength $b > 1$ and $I_0(b)e^{-b} \simeq (2\pi b)^{-\frac{1}{2}}$. Then Eq. (2.19) becomes

$$2 + \frac{\omega_e}{\omega}\left(1 + \frac{k_{\parallel}^2 v_i^2}{2\omega^2}\right)(2\pi b)^{-\frac{1}{2}} + \frac{\beta}{2b}\frac{\omega_e^2}{k_{\parallel}^2 v_A^2} - i\,\frac{\epsilon_0^{-\frac{1}{2}}\omega_e}{2|k_{\parallel}|v_e} = 0. \qquad (2.24)$$

For the Alfvén mode, $\omega_e^2 > 4k_{\parallel}^2 v_A^2$ and the basic assumption $v_i \ll |\omega/k_{\parallel}| \ll v_e$, Eq. (2.24) gives the unstable Alfvén mode

$$\omega = -\frac{\omega_e}{(2\pi b)^{\frac{1}{2}}}\left(2 + \frac{\beta}{2b}\frac{\omega_e^2}{k_{\parallel}^2 v_A^2} - \frac{\epsilon_0^{-\frac{1}{2}}\omega_e}{2|k_{\parallel}|v_e}\right)^{\frac{1}{2}} . \qquad (2.25)$$

For this case, the effect of magnetic shear $k_{\parallel}$ becomes insensitive to stabilize the drift instability of the plasma.

2.4c General Shear Stability Criterion for Convective Modes

To evaluate the spatial growth of a perturbation with frequency ω produced by a narrow source, the form of the relevant approximate solution is

$$\exp\int ik_x(x, \omega)dx$$

where k_x is a wave number and can be found from a dispersion relation. This form is equivalent to the expression of the wave packet (Eq. 2.13). When the amplification reached by the perturbation in a plasma region is less than n_o, the general stability criterion can be expressed as[19,20]

$$-\,\mathrm{Im}\int_o^{x_{max}} k_x(x, \omega)dx < n_o \qquad (2.26)$$

in which $\exp(n_o)$ represents a tolerable level of growth of the convective (or transient) modes.

To illustrate the use of the general stability criterion, Eq. (2.20) and the first of Eqs. (2.23) for the convective Alfvén mode can be taken to advantage. As before, letting $\omega = \omega_2 + \delta\omega_2$ in Eq. (2.20), it follows

$$- \operatorname{Im} k_x = \frac{\epsilon_o^{-\frac{1}{2}}\omega_2}{2ak_{\parallel}^2 v_e v_A} \left(\frac{\omega_e + \omega_2 + \delta\omega_2}{|\omega_2 - \omega_e|}\right)^{\frac{1}{2}} \left[\left(\frac{\omega_e^2}{4} + k_{\parallel}^2 v_A^2\right)^{\frac{1}{2}} \delta\omega_2 + \frac{k_{\perp}}{|k_{\parallel}|} L_s k_{\parallel}^2 v_A^2\right]^{\frac{1}{2}}$$

Integrating this, and considering $\delta\omega_2$ as a parameter, the amplification of the growth of the propagating wave packet becomes

$$- \operatorname{Im} \int_0^{x_{max}} k_x dx = - \frac{\epsilon_o^{-\frac{1}{2}}\omega_e}{2a_i k_{\parallel}^2 v_e v_A} \left(\frac{\omega_e + \omega_2 + \delta\omega_2}{|\omega_2 - \omega_e|}\right)^{\frac{1}{2}} \cdot$$

$$\int_0^r \left[\left(\frac{\omega_e^2}{4} + k_{\parallel}^2 v_A^2\right)^{\frac{1}{2}} \delta\omega_2 + \frac{k_{\perp}}{k_{\parallel}} \frac{r}{L_s} k_{\parallel}^2 v_A^2\right] dr$$

For simplicity, assigning the value $2\omega_e^2 \simeq k_{\parallel}^2 v_A^2$, taking the value of $\delta\omega_2$ corresponding to the amplification, and performing the integration, the stability criterion of the convective Alfvén mode is

$$L_s/r < 6n_o(m_i/\epsilon_o m_e)^{\frac{1}{2}} \qquad (2.27)$$

where

L_s = shearing distance,

$r = -\left(\frac{1}{n}\frac{dn}{dx}\right)^{-1}$ = characteristic or scale length of density gradient (or plasma radius), as defined.

Recalling the required condition $\omega_e^2 > 4k_{||}^2 v_A^2$, the approximation $\omega_2 = \omega_e (k_{||} v_A/\omega_e)^2$ and carrying out the integration, the stability criterion becomes

$$\frac{L_s}{r} < \frac{3}{4} n_o \left(\frac{r}{a_i}\right)^2 \left(\frac{k_y}{k_{||}}\right)^2 \beta \left(\frac{m_i}{\epsilon_o m_e}\right)^{\frac{1}{2}} \tag{2.28}$$

It is important that with high β, the magnitude of magnetic shear needed for stabilizing the convective Alfvén modes is relatively small.

For purposes of comparison, for higher values of magnetic shear and larger values of the relation exists such that $x_1 |\omega/k_{||} v_i| \sim 1$, in the region where the ion Landau damping is strong enough to suppress drift instability. In this case, $\omega \simeq -\omega_e$, by a slight modification of Eq. (2.18), the stability criterion for the electrostatic modes driven by density gradient is

$$L_s/r < 8^{\frac{1}{2}}(r/a_i) \tag{2.29}$$

which is corresponding to the stability criterion obtained by Krall and Rosenbluth[19]

For the localized region of the perturbation extending into the region of the electron Landau damping, Mikhailovskaya and Mikhailovsky obtained, however, another stability criterion[21]

$$L_s/r < (r/a_i)^{\frac{1}{2}} \beta^{-\frac{1}{4}} \tag{2.30}$$

which deals with the oscillations and stability of an inhomogeneous plasma contained in helical magnetic field.

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In the above wave packet analysis, only the density gradient dn/dx, or dn/ndx of the plasma has been considered. If the temperature gradient dT/dx, or dT/Tdx (where $T = T_i = T_e$) is also considered, the parameter, $\frac{dT/Tdx}{dn/ndx} = \frac{d/dx(\ell n\ T)}{d/dx(\ell n\ n)}$, or $\frac{d(\ell n\ T)}{d(\ell n\ n)}$ in the x direction has an important effect on the magnetic shear and plasma stability[15]. In general, for a large ion temperature gradient, no small magnetic shear r/L_s is sufficient to stabilize the temperature gradient instability where $r^{-1} = -\frac{1}{n}\frac{dn}{dx}$ and L_s = characteristic shear distance as defined.

From the wave packet analysis for both density gradient and temperature gradient present in the plasma, the following analytical results can be obtained.

- For a density gradient with a nonparallel electron temperature gradient $d\ \ell n\ T_e/d\ \ell n\ n < 0$, or a density gradient with a small parallel electron temperature gradient, $d\ \ell n\ T_e/d\ \ell n\ n < 2$, a magnetic shear required for plasma stabilization is $r/L_s \sim (m_e/m_i)^{1/3}$.
- For a density gradient with a large parallel electron temperature gradient, $d\ \ell n\ T_e/\ d\ \ell n\ n > 2$, the magnetic shear required for plasma stabilization is $r/L_s \sim (m_e/m_i)^{\frac{1}{2}}$.
- For a density gradient with a nonparallel ion temperature gradient $d\ \ell n\ T_i/d\ \ell n\ n < 0$, or a large parallel temperature gradient $d\ \ell n\ T_i/d\ \ell n\ n > 2$, there is no magnetic shear related to m_e/m_i found for the plasma stabilization.

2.5 Magnetic Well

It has been experimentally demonstrated that universal or interchange instability is considerably inhibited in a thermonuclear plasma contained in a magnetic well, i.e., at a minimum magnetic field — minimum B.[22] Theoretical analysis also predicted that interchange instability of the plasma can be contained in a magnetic well, i.e., at a (on the average) well-shaped magnetic field — minimum average magnetic field or minimum $\bar{B}$.[23,24] The magnetic well may be conceived as the flux surface of magnetic field (or magnetic surface). In other words, the plasma may be contained in a magnetic well where the field strength is increased in every direction away from the plasma. In general, the magnetic well concerns the shape of magnetic field lines.

2.5a Minimum B (Open-End System)

The concept of the minimum B is to stabilize a thermonuclear plasma in a magnetic mirror device of the open-end system. For the interest of a magnetic mirror, if the requirement of self-consistent equilibria and plasma potentials in minimum magnetic fields is satisfied, a stability criterion for moderate curvature of axially-symmetric magnetic surface is given by[25]

$$- \, d\ell \cdot \frac{1}{B^4} \frac{\partial B}{\partial x} \frac{\partial p}{\partial x} > 0 \qquad (2.31)$$

where p is the kinetic pressure, B is the magnetic field of the plasma, and the integration is performed along the element $d\ell$ of the magnetic line.

In an axially symmetric field, it is adequate to take the magnetic flux Ψ for x as the radial coordinate. Hence Eq. (2.31) becomes

$$- \int d\ell \frac{1}{B^4} \frac{\partial B}{\partial \Psi} \frac{\partial p}{\partial \Psi} > 0 \qquad (2.32)$$

Plasma stability is feasible if the pressure is maximum at the minimum B. In other words, when $\partial p/\partial x$ and $\partial B/\partial x$ have opposite signs, Eq. (2.31) or (2.32) is positive and the stability criterion is satisfied.

From plasma energy consideration, a stability criterion related to the particle temperatures and the magnetic well depth for univeral instability can be obtained. If the initial free energy of the plasma is expressed as the Helmholtz function, A, then A(0) is given by[26]

$$A(0) \simeq (nVT_e)(\lambda^2/6r) \qquad \text{at } t = 0 \qquad (2.33)$$

where

$n = n_e$ = electron density,

V = plasma volume,

T_e = electron temperature,

λ = wavelength, width of local instability zone, and

$r = -\left(\frac{1}{n}\frac{dn}{dx}\right)^{-1}$ as defined.

Let the instability be localized in a zone of width λ equal to a typical wavelength. A large number of charged particles diffuse across the field lines. If the plasma expands within the zone $\lambda(dn/dx)$, the change in average energy is $\mu(dB/dx)$ (where μ = constant magnetic moment), $\mu B = (T_i + T_e)$, and the well depth $\nabla B = r(dB/dx)$, then the initial change

in energy, δW, required for disturbances (in which plasma expands across the field in every x direction), is

$$\delta W = \Sigma V \lambda^2 \mu \frac{dn}{dx}\frac{dB}{dx} = -\frac{\lambda^2}{r^2}\frac{\nabla B}{B} nV(T_i + T_e) \tag{2.34}$$

As usual, negative δW implies stability. By combining Eqs. (2.33) and (2.34), the stability criterion for minimum-B to stabilize universal instability at the initial state is

$$\frac{\nabla B}{B} \geq \frac{1}{6(1 + T_i/T_e)} \tag{2.35}$$

provided that the oscillation frequency is less than the ion cyclotron frequency and the wavelength is greater than the ion gyroradius. If $T_e \simeq T_i$, the stability criterion for the depth of the magnetic well required is reduced to

$$\nabla B/B \geq 1/12 \tag{2.35a}$$

if the particle temperatures are not uniform, for simplicity, let the particle temperatures $T_i \simeq x dT_i/dx$ and $T_e \simeq x dT_e/dx$ at any point x of the thin plasma column, then the stability criterion becomes

$$\frac{\nabla B}{B} \geq \frac{1}{6\left(1 + \frac{dT_i}{dx}\Big/\frac{dT_e}{dx}\right)} \tag{2.36}$$

2.5b Minimum Average B (Minimum $\bar{B}$, Closed-End System)

In the toroidal system (closed-end system), it is usually found that plasma particles diffuse away rapidly across the magnetic field lines as in the magnetic mirror system (open-end system). To prevent the universal or interchange instability causing plasma particle leakage, the magnetic-well idea can be utilized by placing toroidal plasma within a deep magnetic well. This idea, however, cannot be done by conduction coils or rods as in the magnetic mirror system.

Although a true toroidal magnetic well is impossible for the closed-end system, the net magnetic well or minimum average B (minimum $\bar{B}$) for the stabilization of a toroidal plasma has been effective.[27,28] The trend of the minimum average B seems toward more sophisticated and more precisely controlled magnetic field configurations.

A magnetic field with helical symmetry can provide a mean or average magnetic well, i.e., provide regions wherein $\int d\ell/B$ decreases away from a magnetic axis (or equivalently $V'' = d^2V/d\Psi^2$ is negative where V is the volume and Ψ is the flux of a magnetic surface enclosing the axis and the volume). The integral can be carried out along the element $d\ell$ of a magnetic field line. In most cases, however, the quantity of the second derivative of the volume per unit flux, V'', is conveniently evaluated in terms of

$$V'' = \int \frac{d\ell}{B} = \int \frac{dz}{B_z} < 0 \qquad (2.37)$$

in which B_z is the axial magnetic field along the z-axis.

Stabilization by negative V'' (i.e., by a favorable gradient of $d\ell/B$ or of average B) is possible in toroidal system and can be more effective than stabilization by magnetic shear against the resistive interchange modes

Minimum average B derived from negative V" of the toroidal system against the universal instability is about the same value as minimum B of the magnetic mirror system.

Magnetic wells with central-rod conductor to carry currents (i.e., hard core) may have three types of the stagnation-point (or stationary-point) solution: (1) Linear periodic-multipole structure, by using helical coils, for instance, $\ell = 2$ and 4 for shaping field with normal stagnation points, and by superimposing $\ell = 1$ and 3 for corrugating field to create favorable ∇B regions of magnetic well. The basic principle in designing a negative V'' system is that along any given magnetic field line the favorable curvature should outweigh the unfavorable curvature; (2) Helical flux equilibrium, by using gross helical curvature to create favorable ∇B regions and by using an axial conductor around which the helical equilibrium flux tube is wound; and (3) Toroidal flux equilibrium, by using gross toroidal curvature to create favorable ∇B regions of the magnetic well. For each solution, a magnetic scalar potential is properly assumed to satisfy the necessary and sufficient conditions of the problem.[29]

Magnetic wells of the helical field without central-rod conductor are also considered. The quantity of the negative V'' is evaluated by assuming some stream functions for magnetic flux.[27] From a practical viewpoint, the central rod is inconvenient.

2.6 Multipole Configuration

Multipole configuration consists of two major types linear multipole and toroidal multipole. The linear multiple device has been operated as a quadrupole, generally in the collisionless regions, with plasma density $n = 10^{10} - 10^{12}\ cm^{-3}$ and temperature 1-10 eV.[30] Plasma loss is

mainly across the magnetic field of the quadrupole. Toroidal multipoles have been studied analytically and experimentally from low β equilibrium and flute instability to high β, ballooning stability.[31-35] The confinement of low density ($n = 10^9$ cm^{-3}) collisionless plasma with $T_i = 40$ eV, $T_e = 10$ eV produced by gun injection or with $T_e = 1$ eV, $T_i < 1$ eV produced by microwave heating was performed to determine the mechanisms for plasma loss from a toroidal multipole magnetic field.[36] The machine has four rods carrying current in one direction and the wall of the device carrying the return current. The plasma loss was attributed to various mechanisms of the mechanical hoops, guard supports, local walls, plasma fluctuation, diffusion, and the generation of low frequency electric fields by the guard obstacles. The chief advantage of the multipole configuration is to study plasma behavior in a magnetohydrodynamic (MHD) stable region or in the $\int d\ell/B$ stable region.

To study the plasma confinement, stability, and fluctuations, the typical magnetic field shape in quadrupole and octapole configurations is shown in Fig. 4. The magnetic field strength distribution in the device (along the central vertical line through the median plane and along the chamber wall) varying with plasma radius is given in Fig. 5. For the quadrupole configuration, the magnetic line of force which forms a cusp, returns around the current-carrying rods. For the octapole configuration, the four rods (with magnetic lines) carrying currents are all in the same direction, and the plasma chamber wall carries the return current. A stable plasma region has its maximum pressure on the lines $\Psi = 0$. The last stable or critical line is Ψ_c. For stability, the plasma pressure must decrease from $\Psi = 0$ to $\Psi = \Psi_c$.

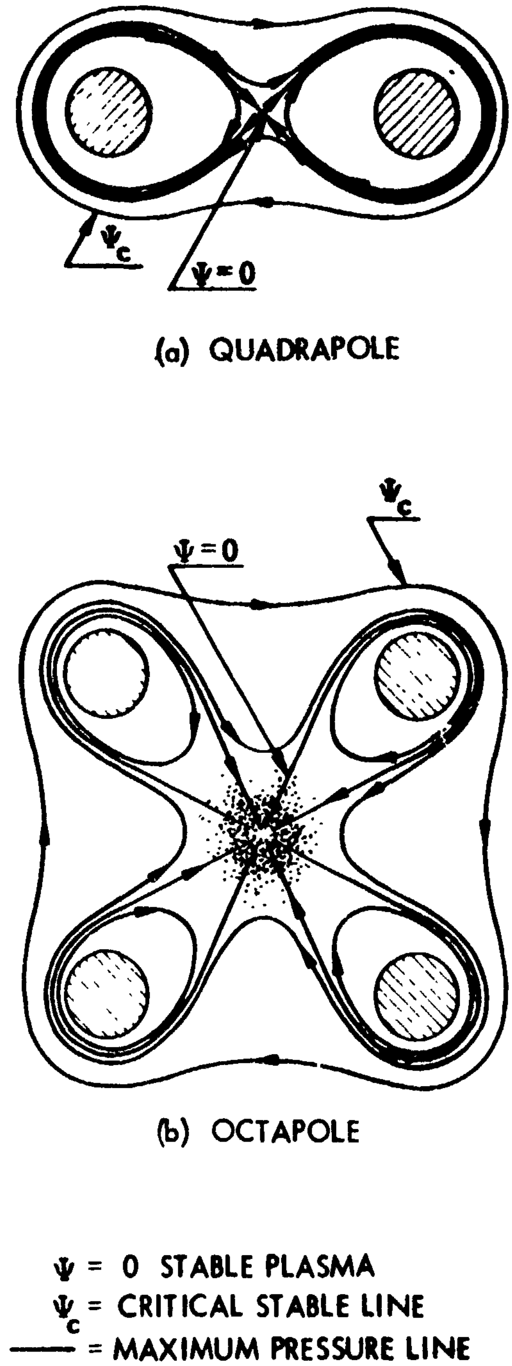

Fig. 4. Typical magnetic field shape in quadrapole and octapole configuration.

As pointed out above, two principles (or methods) are considered important for stabilization: the principle of magnetic shear and the principle of magnetic well (minimum B for the open-end system, and minimum $\bar{B}$ for the closed-end system with short connection length between the favorable curvature and the unfavorable curvature). In the toroidal multiple configurations, the magnetic shear can be varied by superimposing a toroidal magnetic field. The depth of the magnetic well and the connection length can be altered by changing the number of poles, for instance, from octapole to quadrupole.

The superposition of a toroidal field produces both magnetic shear and finite parallel wave numbers. In case the toroidal field B_t is much smaller than the average multipole field B_m, the parallel wave number $k_{\parallel}$ is approximately given by $k_{\parallel} \approx (B_t/B_m)k_{\perp}$ where $k_{\perp}$ is the perpendicular wave number. The effect of the magnetic shear tends to stabilize the plasma in a multipole configuration.

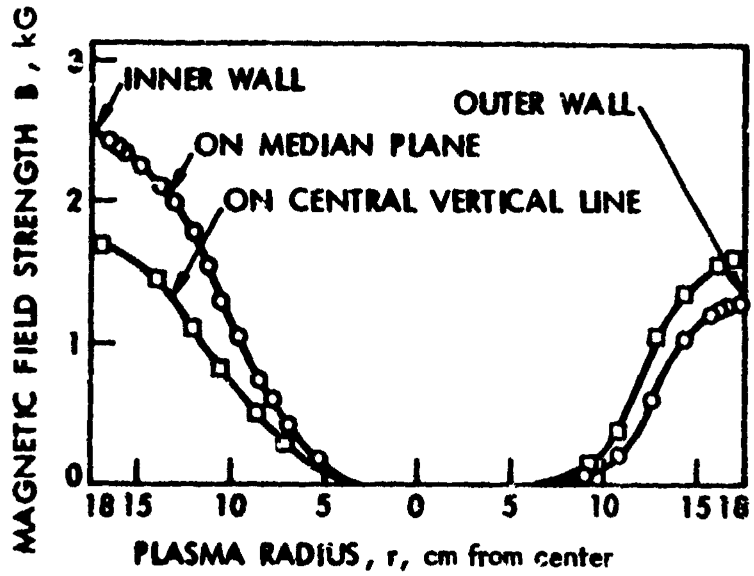

Fig. 5. Magnetic field strength distribution in the device vs plasma radius.

The magnetic well depth of a toroidal multiple configuration can be defined as $\left\{\left(\int d\ell/B\right)_{\Psi=\Psi_c/2} - \left(\int d\ell/B\right)_{\Psi=\Psi_z}\right\} \Big/ \left(\int d\ell/B\right)_{\Psi=\Psi_z}$ where Ψ is the magnetic flux function and Ψ_c is the flux at the critical line (see Fig. 4). The magnetic well of the octapole configuration is usually much deeper than that of the quadrupole. The flux surface where the magnetic shear vanishes is closed to the $\Psi = \Psi_c$ surface when the aspect ratio of the device is sufficiently large.

In the toroidal multipole devices, drift cyclotron instability and interchange (or universal) instability have been identified.[37] The corporation of the magnetic shear and the magnetic well with short connection length between favorable and unfavorable regions into the multipole configuration may improve the plasma stability.

3. DYNAMIC STABILIZATION

The purpose of dynamic stabilization is to perturb the equilibrium parameters of a whole thermonuclear plasma to stabilize it. This can be accomplished analytically or experimentally by assuming (or applying) an oscillating magnetic field, oscillating electric field, or microwave heating. The dynamic stabilization is effective, which can stabilize many modes of one instability and other types of instabilities simultaneously.[39,40]

The dynamic stabilization of a thermonuclear plasma may consist of the following proposed methods: (a) Traveling wave and standing wave, (b) oscillating electromagnetic field, (c) oscillating electric field, (d) electron temperature modulation, and (e) axial alternating current. Other methods of dynamic stabilization may be developed for certain purposes or circumstances once the basic behavior of stabilization mechanisms is better understood.

3.1 Traveling Wave and Standing Wave

The dynamic stabilization of linear θ pinch has been previously proposed.[40] In the model under consideration the collisionless plasma consists of charged particles which can be reflected into it from its surface. Every perturbation of the plasma gives rise to an excessive average magnetic pressure which opposes the unstable deformation on the plasma surface. In addition to the axial magnetic field, the application of a periodic constant azimuthal magnetic field along the pinch axis can produce a significant effect on the $m = 1$ instability of the plasma.

The dynamic stabilization of the $\beta = 1$ bumpy or toroidal θ pinch has also been studied.[41] In the analytical study, the dynamic stabilizing currents are applied perpendicular to the axial magnetic field. (Without the dynamic stabilization, the configuration is basically unstable.) The plasma is assumed incompressible because in the analysis of the static pinch the most unstable modes of the plasma are found incompressible. The new concept introduced in the analysis is to make the corrugated magnetic field travel along the pinch axis at the wave velocity v_w (where $v_w > v_A$ and v_A = Alfvén velocity) because the outward drift of the plasma due to the curvature of the torus can be suppressed by corrugating the plasma surface. In other words, the corrugated magnetic field and surface profile are made to propagate along the pinch axis with a wave velocity v_w by suitably alternating part of the current in the conduction coils. This concept is referred to as "dynamic stabilization of the plasma by traveling wave" (instead of standing or stationary wave at steady state). It may be assumed that the frequency of the magnetic field should be greater than v_w/λ with the traveling wave and less than v_s/λ with the standing wave where v_s is the sonic velocity or the mean ion velocity and λ is the wavelength.[42] Dynamic stabilization by traveling wave is more effective than that by standing wave, in some cases. The idea of the dynamic stabilization of a thermonuclear plasma either by the traveling wave or by the standing wave depends basically on the equilibrium principle that all resultant destabilizing forces at any point of the plasma (contained in a linear or toroidal system) must be vanished.

3.2 Oscillating Electromagnetic Field

The desirability of stabilizing the universal instability of an inhomogeneous plasma in a magnetic field by dynamic stabilization has been studied.[40-43] The application of an oscillating electromagnetic field in stabilizing plasma is briefly analyzed in this section. Consider a collisionless plasma confined in the x direction by a uniform magnetic field B_z, parallel along the z axis. The scale length of density gradient is given by $r^{-1} = -\, dn/ndx$ where n is the plasma density (n decreasing with increasing x). The temperature gradient of the plasma is small conpared to the density gradient. An oscillating electromagnetic field which consists of the electric field E and the magnetic field B can be expressed in terms of the scalar potential ϕ and the vector potential $\vec{A}$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t} \qquad \vec{B} = \nabla \times \vec{A} \tag{3.1}$$

where

$\vec{A} = a_o(e^{i\theta}\vec{a}_1(x) + \ldots)$,

$\phi = a_o(e^{i\theta}\phi_1(x) + \ldots) + a_o^2\phi_o(x)$,

$\theta = k_y y + k_z z + \Omega t$,

Ω = the frequency of the electromagnetic field,

c = the speed of light,

k_y, k_z = wave numbers along the y and z axes, respectively,

$\phi_o(x)$, $\phi_1(x)$ = given functions, and

a_o = amplitude of the electromagnetic field and is assumed to be small, and $\vec{a}_1(x)$ are given vectorial functions.

The stabilizing effect produced by the electromagnetic field is due to the drift velocity of the guiding center of ions and electrons. The dynamic stabilization is effective when the drift velocity is of the order of the diamagnetic ion velocity v_{di} (= negative diamagnetic electron velocity v_{de}, where $v_{di} = T_i/m_i r\omega_{ci}$, $T_i = T$ = ion temperature, m_i = ion mass and ω_{ci} = ion cyclotron frequency).

In the absence of the oscillating electromagnetic field, let ρ_c be the radius of gyration, $v_{||}$ the velocity parallel to the z-axis, and x_c the abscissa of the guiding center of a charged particle. The motion of the particle is not in resonance with the electromagnetic field.

In the presence of the electromagnetic field, the motion of the charged particle at a given point x is specified as

$$\bar{v}_y t + f_y(\tau_1, \tau_2) \text{ and } \bar{v}_z t + f_z(\tau_1, \tau_2)$$

where

$$\tau_1 = \bar{\omega}_c t,$$

$$\tau_2 = (\Omega + k_y \bar{v}_y + k_z \bar{v}_z)t,$$

$\bar{v}_y$, $\bar{v}_z$ and $\bar{\omega}_c$ = the mean velocities in the y and z directions and the mean cyclotron frequency of the particle, respectively,

t = time, and

$f(\tau_1, \tau_2)$ = the periodic function.

For given time t and particle energy W, the mean velocities and the mean cyclotron frequency of the particle as functions of ρ_c, $v_{||}$, x_c can be given by

$$\bar{v}_y(\rho_c, v_{||}, x_c) = \frac{1}{m\omega_c}\frac{\partial W}{\partial x_c}$$

$$\bar{v}_z(\rho_c, v_{||}, x_c) = v_{11} + \frac{1}{m}\frac{\partial W}{\partial v_{||}} \tag{3.2}$$

$$\bar{\omega}_c(\rho_c, v_{||}, x_c) = \omega_c - \frac{2}{m\omega_c}\frac{\partial W}{\partial \rho_c^2}$$

where $\omega_c = \frac{qB_z}{m}$, q = charge, and m = mass of the particle.

For an inhomogeneous plasma with uniform temperature T, the scalar potential ϕ_T may be written as

$$\phi_T = b(x, \theta)e^{i(k_y y + k_z z + \omega t)}$$

in which $b(x, \theta)$ is a function of x and θ, periodic nature in θ with a period 2π. Comparing the expressions given for Eq. (3.1), $b(x, \theta)$ can be expanded into series as

$$b(x, \theta) = b_o(x) + b_1(x)e^{i\theta} + b_{-1}e^{-i\theta} + \ldots \simeq b_o(x)$$

where $b_1(x)$, $b_{-1}(x)$, ... are small compared with $b_o(x)$ and are of the order of magnitude corresponding to the small amplitude, a_o.

For drift modes of low frequency $|\omega| \ll |\omega_{ci}|$, $b_o(x) = h(x)e^{ik_x x}$ where h(x) is defined by the condition $|dh/hdx| < 1/4r_i$ (where $r_i = [2T_i/m_i\omega_{ci}^2]^{\frac{1}{2}}$ = mean radius of gyration of ions), the dispersion relation of the plasma oscillations in the presence of the oscillating electromagnetic field is given by[44]

$$L_i + L_e + Da_o^2 = 0 \tag{3.3}$$

where $L(L_i$ and L_e for ions and electrons) is related to the mean value of the particle

$$L = 1 - \frac{(\omega + k_y v_d) J_o^2(k_\perp \rho_c)}{\omega + k_y \bar{v}_y + k_z \bar{v}_z} \frac{\iiint f(\rho_c, v_{||}) h^2(x_c) d\rho_c^2 dv_{||}\, dx_c}{\iiint f_o(\rho_c, v_{||}) h^2(x) d\rho_c^2 dv_{||}\, dx_c}$$

where

$f_o(\rho_c, v_{||})$ or $f(\rho_c, v_{||}) \simeq$ the Maxwellian distribution $\exp\left[- m(\omega_c^2 \rho_c^2 + v_{||}^2)/2T\right]$

$$k_\perp^2 = k_x^2 + k_y^2$$

$J_o(k_\perp \rho_c)$ = the Bessel function of zero order of the first kind, and

D = real and bounded in absolute value independent of the modes.

The plasma is dynamically stabilized if the real parts $\mathrm{Re}L_i > 0$ and $\mathrm{Re}L_e > 0$ when $|Da_o^2| \ll 1$, $|\omega| < |k_y v_d|$, and $|dh/hdx| < 1/4r_i$. These conditions can be satisfied if the values of function $v_y(\rho_c, v_{||}, x_c)$ for each set in the domain, $\rho_c^2 < 2T/m\omega_c^2$, $v_{||}^2 < T/m$ and $|x_c| < r_i$ are spread over an interval of the order of $|v_d|$ ($= \frac{T}{m\omega_c r}$ = diamagnetic velocity).

There are two cases related to the dynamic stabilization by the standing wave of the oscillating electromagnetic field:

(a) when the oscillating electromagnetic field is an electromagnetic standing wave along the x-axis, then

$k_y = k_z = 0$, $|\omega_{c_e}| \ll \Omega \gg k_x (T/m_e)^{\frac{1}{2}}$ and $\Omega \gg \omega_p$ ($\omega_p = (4\pi n q^2/m_e)^{\frac{1}{2}}$ = the plasma frequency).

Let $\phi_1(x) = 0$, $\vec{a}_1(x) = 2\vec{b} \cos k_x x$, and $c^2 k_x^2 = \Omega^2 - \omega_p^2$, for the electromagnetic field induced by an external source, it follows that

$$\phi_o(x) = - \frac{b^2 q}{m_e c^2} \cos k_x x.$$

Hence Eqs. (3.2) are reduced to

$$\overline{v}_y(\rho_c, v_{\|}, x_c) = \frac{k_x}{m\omega_c} \frac{q^2\overline{E^2}}{2m_e\Omega^2} J_o(2k_x\rho_c) \sin 2k_x x_c$$

where $\overline{E}^2$ is the mean square intensity of the electric field of the oscillating electromegnetic field.

(b) when the oscillating electromagnetic field is an electrostatic standing wave along the x-axis: as before, let $k_z = a_1(x)$ $= \phi_o(x) = 0$

$|\Omega/\omega_{ci}| \ll |k_y/k_x| \ll 1$, and $\phi_1(x) = 2b \cos k_x x$

If the electromagnetic field is produced in the plasma, then Eqs. (3.2) become

$$\overline{v}_y(\rho_c, v_{\|}, x_c) = \frac{2q^2\overline{E^2}k_y}{m\omega_c^2\Omega} J_o^2(k_x\rho_c) \cos 2k_x x_c .$$

3.3 Oscillating Electric Field

Oscillating high-frequency electric field applied for dynamic stabilization of an inhomogeneous plasma consists of two cases:

(a) Uniform high-frequency electric field, when the skin laver $\delta_\perp$ of the field is equal or greater than the transverse dimensions of the plasma $L_\perp$, i.e., when $\delta_\perp \gtrsim L_\perp$, the electric field is assumed uniform.

(b) Non-uniform high-frequency electric field, when the skin layer depth $\delta_\perp < L_\perp$, the electric field is considered non-uniform in the plasma.

In both cases the stabilizing effects depend on the plasma parameters and the frequency and strength of the electric field.

A thermonuclear plasma confined by a magnetic field in a closed-end device is subject to both hydromagnetic and drift instabilities. These instabilities can result in anomalous diffusion of the plasma across the magnetic field. High-frequency dynamic stabilization of hydromagnetic instability has been studied;[40-45] dynamic stabilization of drift (gross or universal) instability by oscillating electric field has also been considered.[46-48]

3.3a Uniform High-Frequency Electric Field

The stabilization of collisionless drift instability in an inhomogeneous plasma is feasible by the application of a uniform high-frequency electric field E(t) in the direction parallel to the magnetic field, which may be called the "E-stabilization":

$$E(t) = E_0 \cos \Omega t \tag{3.4}$$

where $E(0) = E_0$, t is the time and Ω is the frequency of the applied electric field. When a high-frequency electric field of sufficient amplitude operates in the plasma, the oscillations of electrons in the field become important. If a drift instability is excited in the plasma, the plasma electrons are subject to an additional force in the direction parallel to the magnetic field due to the presence of the high-frequency electric field, $E(t)$ and $E_1(t_1, x)$. This force is equal to the electron charge times the mean values of the electric field, $e(\overline{E} + E_1)$, where E_1 is the part of high-frequency induced electric field. The parameter $d(\ln T)/d(\ln n)$ plays a very important role in the determination of stabilizing effect for different frequency regions related to the parameter of ion speed ratio, ζ_i (here T = plasma temperature and n = plasma density as defined).

When this additional force is in phase with the force due to kinetic pressure of the plasma, the electric field in the drift wave is increased and so is the frequency, ω. Under the conditions $(T_i/m_i)^{\frac{1}{2}} \ll \omega/k_i \ll (T_e/m_e)^{\frac{1}{2}}$ and $\zeta_i = (k_\perp/\omega_L)(2T_i/m_i)^{\frac{1}{2}}$ of the intermediate-frequency case ($k_\perp^2 = k_x^2 + k_y^2$, ω_L = Larmor frequency), as ω is increased, the Landau damping by the electrons is intensified. Therefore the collisionless drift instability of the plasma tends to be stabilized. The range of values of the parameter $d(\ell n T)/d(\ell n\ n)$ in which high-frequency stabilization occurs is maximal for $\zeta_i \to 0$ and approaches zero when $\zeta_i \to \infty$. For the low-freqeuncy case, $\omega \ll k_z(T_i/m_i)$ the instability region is the same as in the absence of high-frequency electric field $d(\ell n\ T)/d(\ell n\ n) > 2(1 + \delta)$ where $\delta = \zeta_i^2\left[1 - I_1(\zeta_i^2/2)/I_o(\zeta_i^2/2)\right]$ and $I_o(\rho_i^2/2)$, $I_1(\rho_i^2/2)$ are the modified Bessel functions.

For the high-frequency case, $\omega \gg k_z(T_e/m_e)^{\frac{1}{2}}$, if the conditions $\omega \ll k_o\zeta_i(T_i/m_i)^{\frac{1}{2}}$ and $k_z \ll k_o\zeta_i(m_e/m_i)^{\frac{1}{2}}$ are given and the requirement of the parameter $d(\ell n\,T)/d(\ell n\,n) < (1/\zeta_i - 1)$ is satisfied, the dynamic stabilization of a collisionless inhomogeneous plasma by an oscillating uniform high-frequency electric field with modest amplitude can be achieved. The value of k_o is defined as $k_o = (n_oT)^{-1}d(n_oT)/dx$, where n_o and T are the plasma density and kinetic temperature at equilibrium state.

The electron-ion collisions in a collisional, inhomogeneous plasma resulting in the excitation of drift instability is known as the drift-dissipative instability.[49] When a dispersion relation is derived on the basis of the usual hydrodynamic equations, the collision term $m_e v_e \nu$ due to electron-ion collisions must be taken into account in the equation of electron motion parallel to B_z (where m_e is the electron mass, v_e is the electron velocity and ν is the electron-ion collision frequency in the z direction). It can be shown from solutions of the dispersion

equation that the drift-dissipative instability can also be stabilized by a high-frequency electric field. The amplitude of the uniform high-frequency electric field required, however, is higher than that for the collisionless drift instability obtained.[47]

3.3b Non-Uniform High-Frequency Electric Field

As discussed, if the skin layer depth $\delta_\perp$ of the high-frequency electric field is comparable with the transverse dimension $L_\perp$ or plasma radius, the high frequency electric field can be considered as uniform and the effect of the magnetic field can be neglected[47]. If $\delta_\perp \ll L_\perp$ the high-frequency field skin effect, however, must be taken into account The high frequency electric field is then non-uniform and the effect of the magnetic field component becomes important. In such case, the stabilizing effect of the non-uniform high-frequency electric field gives rise to two additional independent mechanisms of stabilization[48]: (a) the gravitational mechanism produced by the averaged high-frequency field forces in an unperturbed plasma state, (b) the oscillating magnetic-field mechanism of the time-dependent component of the wave field vector projected in the axial magnetic field B_z direction. The gravitational mechanism of stabilization may be called the "f-stabilization," and the oscillating magnetic field mechanism of stabilization may be called the "B-stabilization." Thus, the effects of the B-stabilization, E-stabilization and g-stabilization of drift waves by a non-uniform high-frequency electric field can occur concurrently on a weekly inhomogeneous plasma confined in a constant, main magnetic field, B_{zo}.

Consider a collisionless, inhomogeneous plasma confined in a strong constant magnetic field B_{zo} along the z-axis. The applied non-uniform high-frequency electric field can resolve into two components:

$$E_z = E_o(x) \sin \Omega t, \text{ and}$$
$$B_z = -\frac{c}{\Omega} \frac{\partial E_o(x)}{\partial x} \cos \Omega t \qquad (3.5)$$

where $E_o(x)$ is the amplitude of the applied electric field E, B_z is the induced magnetic field component parallel to the main magnetic field B_{zo}, and Ω, t and c are as defined before. The plasma density n and temperature T vary with x radially.

The averaged high-frequency field force F_α which influences the g-stabilization in an unperturbed plasma state is given by

$$F_\alpha = m_\alpha g_\alpha = - m_\alpha \nabla \Psi_\alpha = - m_\alpha \nabla \frac{q^2 E_{zo}^2(x)}{4 m_\alpha^2 \Omega^2} \qquad (3.6)$$

where

g = the gravitational acceleration,

Ψ = the high-frequency potential function,

E_{zo} = $E_o(x)$ in the z direction, and

α = e, i for electrons and ions in the plasma respectively.

The high-frequency field forces act mainly with the negative electrons. In the steady state, however, there is charge separation together with a potential electric field $E = - \nabla\phi$ (see Eqs. 3.1), or $E_x = - \partial\phi/\partial x$. As a result, the positive ions are also contained.

The conditions of a collisionless, inhomogeneous plasma require that the dimensions of particle density gradient, $r^{-1} = - \frac{1}{n}\frac{dn}{dx}$, and

particle temperature gradient, $(r^{-1})_T = -\frac{1}{T}\frac{dT}{dx}$, and the skin layer depth $\delta_\perp$ of the high-frequency electric field, are large compared to the Larmor radius of the ions.

From the oscillating electric and magnetic fields and the field force produced by the non-uniform high-frequency electric field and the conditions required for a collisionless, inhomogeneous plasma confined in a constant magnetic field, the dispersion relation can be derived and solutions of the dispersion equation can be obtained. In the absence of a high-frequency electric field, the plasma is stable only against a fast ion acoustic wave when the parameter $d(\ell n T)/d(\ell n n) > 0$. The plasma is unstable for $d(\ell n T)/d(\ell n n) < 0$. When the high-frequency electric field is applied, the stable region broadens toward negative values of $d(\ell n T)/d(\ell n n)$. The effect of stabilization is the increased frequency ω which accompanies the applied high-frequency electric field. as a result, the Landau electron damping is intensified. Thus, the stabilizing effect produced by the high-frequency field pressure is associated with the Doppler shift due to drift velocity $v_d = \frac{g_\alpha}{\omega_\alpha}$ (where $\omega_\alpha = qB_z/m_\alpha$) of the charged particles, i.e., the mechanism of the g-stabilization.

In general, the stability of a collisionless, inhomogeneous plasma stabilized by a non-uniform high-frequency electric field depends on the plasma parameters and the frequency and strength of the electric field. An effective stabilization of the plasma by a non-uniform electric field may be achieved near the resonance condition.

Analytical work to predict the effects of dynamic stabilization on drift instability of an inhomogeneous plasma by an oscillating electric field parallel to the magnetic field has been discussed above. Very

recently, experimental work to observe the effect of dynamic stabilization of the plasma by the oscillating electric field has also been reported.[50,51] In the experimental work reported, dischage-type helium plasma and Q-machine cesium plasma have been used. Plasma diagnostics are made by means of radially movable electrostatic (or Langmuir), and magnetic probes. Electron density, electron temperature, magnetic field strength, modulation frequency, wave amplitude, ac voltage, etc., are measured. The decreases of drift wave amplitude and wave spectrum in several modes (m = 1, m = 2, ...) prove the stabilizing effect of the electric field at relatively low and high frequency regions.

A proposed schematic diagram for dynamic stabilization of a discharged plasma by a high-frequency electric field is shown in Fig. 6. Basically, the system consists of two cathodes (with a tubular anode between them) in a magnetic field parallel to the axis of the anode

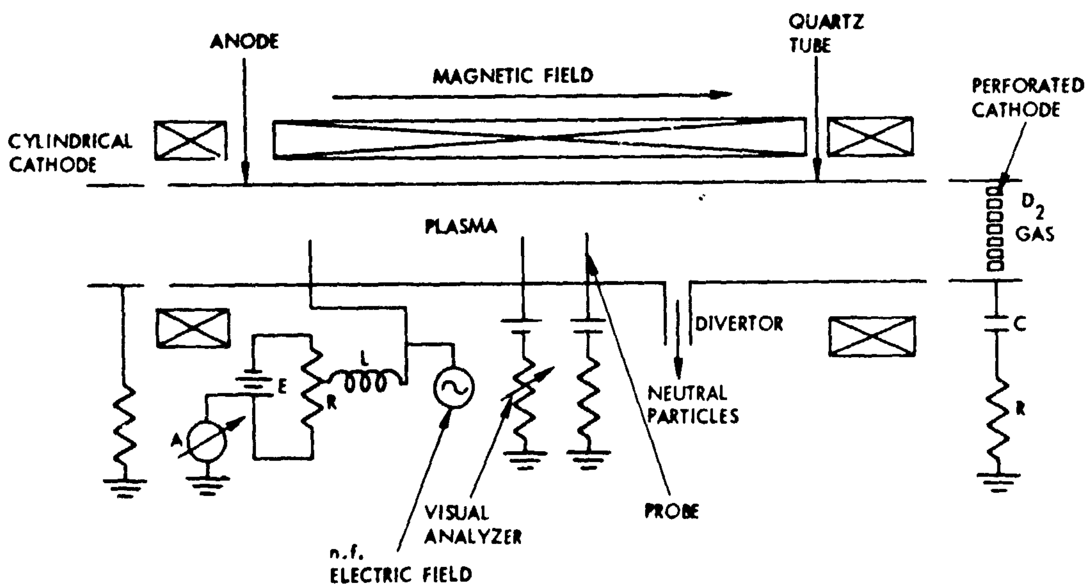

Fig. 6. Schematic diagram for dynamic stabilization of a discharged plasma by high-frequency electric field.

tube. Deuterium gas is discharged through perforated tantalum cathode at the right, and plasma emerges from a tabular cathode at the left. Ions produced in the discharge are accelerated back and forth between the two cathodes, where the ions produce secondary electrons. These electrons are prevented by the magnetic field from reaching the anode, and they in turn produce more ions in the plasma. Neutral particles are removed by the divertor. A sinusoidal ac potential with variable amplitude and frequency is applied near the left end of the plasma column. The average current of the plasma is measured by ammeter A, Fig. 6. Plasma diagnostics can be made by means of movable electrostatic probe, magnetic probe, spectroscopic measurement, visual analyzer and microwave techniques.

3.4 Electron Temperature Modulation

The effect of a small, periodic, time-dependent modulation of the electron temperature on the drift waves of a collisionless, inhomogeneous plasma is considered. The energy source required for electron temperature modulation can be a periodic, intensity-varying microwave to heat the plasma. If the transit time of a particle through the plasma device is short compared to the modulation period of microwave heating, the electron temperature will follow the modulation of the energy source. Energy transfer from the electrons to the ions in the plasma will be small if the electron-ion equipartition time is long compared to the modulation period. Ohterwise, the energy transfer from the electrons to the ions will be large.

If the plasma has a Maxwellian velocity distribution, the modulation electron temperature, $T_e(x, t)$, may be expressed as

$$T_e(x, t) = T_{eo}(1 + a \cos \Omega_m t) \tag{3.7}$$

where T_{eo} is the peak electron temperature, a is the parameter $a \ll 1$, and Ω_m is the modulation frequency of the energy source such as microwave heating. By using the Vlasov equation of electron motion and giving a small perturbation of the equilibrium parameter, a dispersion relation can be derived.[52] For the plasma with weak density gradient, temperature gradient and/or axial current in the limit of vanishing Larmor radius, the stability criterion for different modes of drift wave can be given by[52,53]

$$a^2 > \frac{2\pi(\omega_e^*)^2}{k_z^2 v_e v_i} \left[\frac{|k_z| u}{\omega_e^*} \left(\frac{d\ \ell n\ T_{eo}}{d\ \ell n\ n} - 2 \right) - \frac{d\ \ell n\ T_{eo}}{d\ \ell n\ n} \left(1 - \frac{1}{2} \frac{d\ \ell n\ T_i}{d\ \ell n\ n} \right) \right] \tag{3.8}$$

where

$$\omega_e^* = \frac{k_y v_e^2}{2|\Omega_{ce}|} \frac{1}{n} \frac{dn}{dx},$$

$$v_e = (2\ T_{eo}/m_e)^{\frac{1}{2}},$$

$$v_i = (2\ T_i/m_i)^{\frac{1}{2}},$$

$$\Omega_{ce} = qB/m_e,$$

u = macroscopic electron velocity, $u \ll v_e$,

k_y, k_z = the wave numbers in the y and z directions, and

T_{eo}, T_i, $n(n_e = n_i)$, etc., = previously defined.

Equation (3.8) is valid as long as a $\ll 1$ and $v_i \ll |\frac{\Gamma}{k_z}| \ll v_e$. It is seen that the plasma parameters, $d(\ln T_{eo})/d(\ln n)$ and $d(\ln T_i)/d(\ln n)$ remain to play the important role to stabilize the drift waves by electron temperature modulation.

The theoretical results show that the dynamic stabilization of drift waves by electron temperature modulation for a collisionless inhomogeneous plasma is possible if the stability criterion Eq. (3.8) is satisfied and one of the beat frequency, $\omega_{b_\pm} = \pm \Omega_m - \omega_e^*$ is of the order of $|k_z| v_i$ where the ion Landau damping becomes intensified.

3.5 Axial Alternating Current

Dynamic stabilization of collision-dominated drift waves by means of an axial alternating current (or ac current) is studied. With the collision-dominated region, an inhomogeneous plasma has a uniform magnetic field $B(= B_z)$ in the z direction and the density n and the temperature T varying in the x direction. When an axial electric voltage is applied to the grid of the plasma tube in the presence of an ac current, the drift waves of the plasma at low frequencies can be suppressed.[54,55]

Drift waves in a cesium plasma are stabilized experimentally by applying an external, ac current parallel to magnetic fields through a meshed grid immersed in the plasma.[54] The results of the dynamic stabilization shown in the experimental data may be interpreted by the facts: (a) The bias effect due to rectifying characteristics of a plasma sheath when ac voltage or current is applied to the grid, causes the electron current to increase because of rectifying characteristics of the electron-rich plasma sheath, and (b) some non-linear

Landau damping effect due to couplings between drift waves and external perturbations of the applied current when the applied current frequency Ω is closed to drift wave frequency, and strong coupling between them is observed.

The simple, analytical result for dynamic stabilization of collision-dominated drift waves in the presence of an ac current is also obtained when the applied current frequency Ω is comparable with drift wave frequency.[55] A macroscopic electron velocity $u_o(x, t)$ parallel to the magnetic field B_z can be produced by the ac current flowing in the plasma. From the equation of electron motion, the linearized equation of continuity, the energy balance equation of the system, and simplification, the stability condition for low-frequency drift waves is given by

$$\frac{3u_o^2}{v_{eo}^2}\frac{d\,\ell n\,T_{eo}}{d\,\ell n\,n_o}\left[\frac{3b\nu^2}{k_z^2 v_{eo}^2}\frac{d\,\ell n\,T_{eo}}{d\,\ell n\,n_o} - \frac{T_{io}}{T_{eo}}\frac{d\,\ell n(n_o u_o)}{d\,\ell n\,n_o}\right]$$

$$- b\left(1 + \frac{d\,\ell n\,T_{eo}}{d\,\ell n\,n_o}\right)\left(1 + \frac{T_{io}}{T_{eo}} + \frac{d\,\ell n\,T_{io}}{d\,\ell n\,n_o}\right) > 0 \qquad (3.9)$$

where

$b = \frac{1}{2}k_\perp^2 a_i^2 = \frac{1}{2}(k_x^2 + k_y^2)a_i^2$,

a_i = the ion gyramagnetic radius,

$v_e = (2\ T_e/m_e)^{\frac{1}{2}}$,

ν = the electron-ion collision frequency, $\nu \gg |k_z| v_{eo}$, and

o = the initial or unperturbed state of the plasma.

It is seen from Eq. (3.9) that the combined effects of finite ion Larmor radius, the temperature gradient and density gradient of ions and electrons, the current gradient, and the electron inertial influence are all involved in the stability condition of collision-dominated drift waves.

4. FEEDBACK STABILIZATION

Plasma stabilization by feedback methods has been introduced quite recently.[56-68] The selected and proposed feedback methods to stabilize a thermonuclear plasma are: (a) electrostatic feedback; (b) neutral atom beam feedback; (c) modulated microwave energy sources, and (d) modulated uniformly distributed electron sources. Some new concepts of oscillating or helical magnetic fields for feedback stabilization of a thermonuclear plasma can be developed similarly.

4.1 Electrostatic Feedback

Feedback stabilization consists of sensing the plasma instability at any point in the plasma by sensor devices and then feeding back an accurate signal of proper amplitude and phase in order to suppress the instability. A number of feedback stabilization experiments have been conducted with electrostatic (or Langmuir) probes as sensors as well as control elements. Such experiments have proved successful in suppressing collisional drift instability of the plasma. The theory of electrostatic feedback stabilization of drift waves to interpret the experimental results has also been advanced.[56-60]

Various electrostatic feedback techniques have been developed for plasma stabilization. By modulating a parallel-electron-current sink, for example, (i.e., modulating and drawing electron current from an electrostatic probe parallel to the axial magnetic field) the collisional drift wave is stabilized with negative feedback in the unstable region of the potassium plasma at $T = 0.23$ ev, $n_o = (10^{10})\text{cm}^{-3}$, $dn_o/n_o dx = -1.25\ \text{cm}^{-1}$, and the critical magnetic field $B_c = 2.19$ kilogauss (kG). The drift instability is the $m = 2$ azimuthal mode. The experimental results obtained

for constant feedback gain varying with phase (angle) shift of the applied probe (suppressor) voltage for various magnetic fields are shown in Fig. 7(a-c). Figure 7(a) shows the plasma density or instability amplitude, Fig. 7(b) gives the change of plasma density at the center of the plasma column, and Fig. 7(c) shows the instability frequency as functions of feedback phase shift in both stable and unstable regions respectively. The theoretical instability growth rate and frequency of the plasma are also shown in the curves of Fig. 7(a) and (c). The feedback gain varying with the plasma density, characteristic length of the density gradient and instability frequency of the plasma is shown in Fig. 8(a-c). The radial distribution of equilibrium plasma density n_o, plasma density or instability amplitude n, and required minimum probe (suppressor) voltage V for feedback stabilization is given in Fig. 9. The plasma density at the center of the plasma column as a function of magnetic field strength with and without feedback stabilization of collisional drift waves is shown in Fig. 10.

With a suitable feedback circuit arrangement, stabilization of the m = 2 aximuthal mode can be achieved up to $B = 1.2B_c$ (B_c = critical magnetic field strength). Thus, the electrostatic feedback (negative or positive) demonstrates that collisional drift instability driven by density gradient can be controlled from penetration of feedback signals into the plasma interior with phase shift.

4.2 Neutral Atom Beam Feedback

The application of feedback stabilization by means of electrostatic probe or material suppressor is limited to plasmas of moderate density and temperature because an electrode cannot resist very high temperature

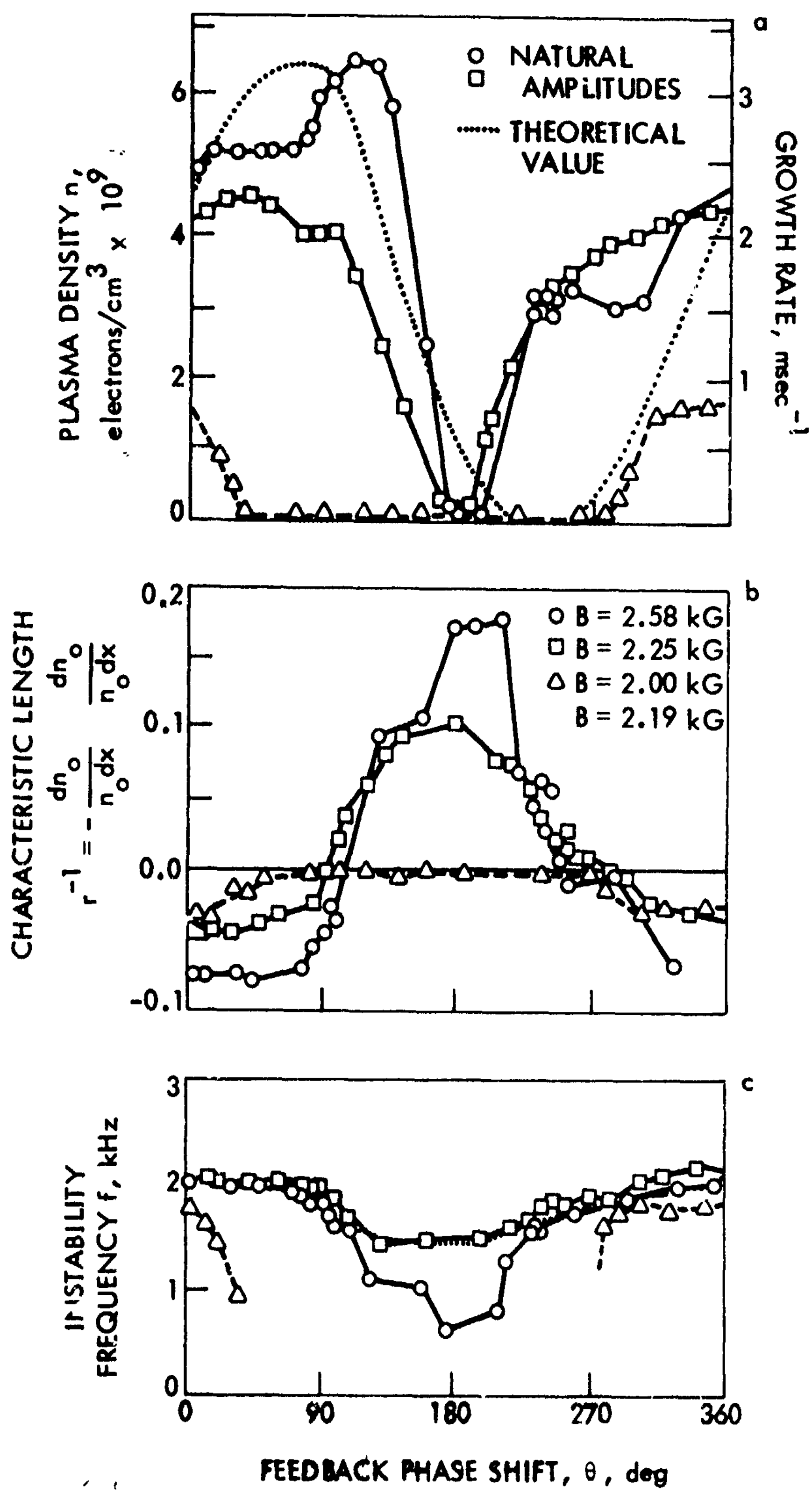

Fig. 7. Constant feedback gain v. phase shift of the applied probe (suppressor) voltage for various magnetic fields: (a) instability amplitude; (b) change of density at center of plasma column; and (c) instability frequency after subtraction of Doppler shift due to radial electric field.

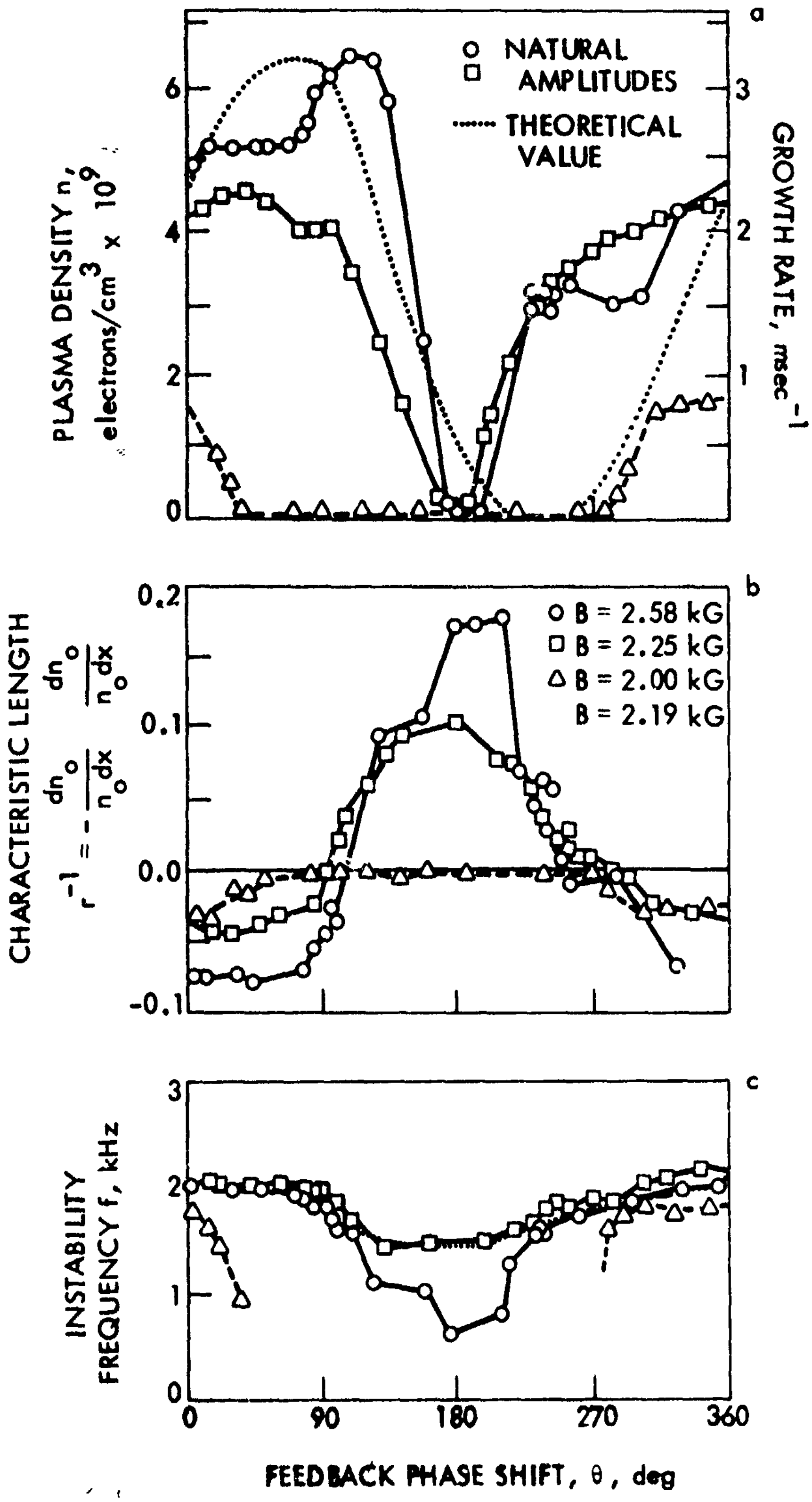

Fig. 7. Constant feedback gain vs phase shift of the applied probe (suppressor) voltage for various magnetic fields: (a) instability amplitude; (b) change of density at center of plasma column; and (c) instability frequency after subtraction of Doppler shift due to radial electric field.

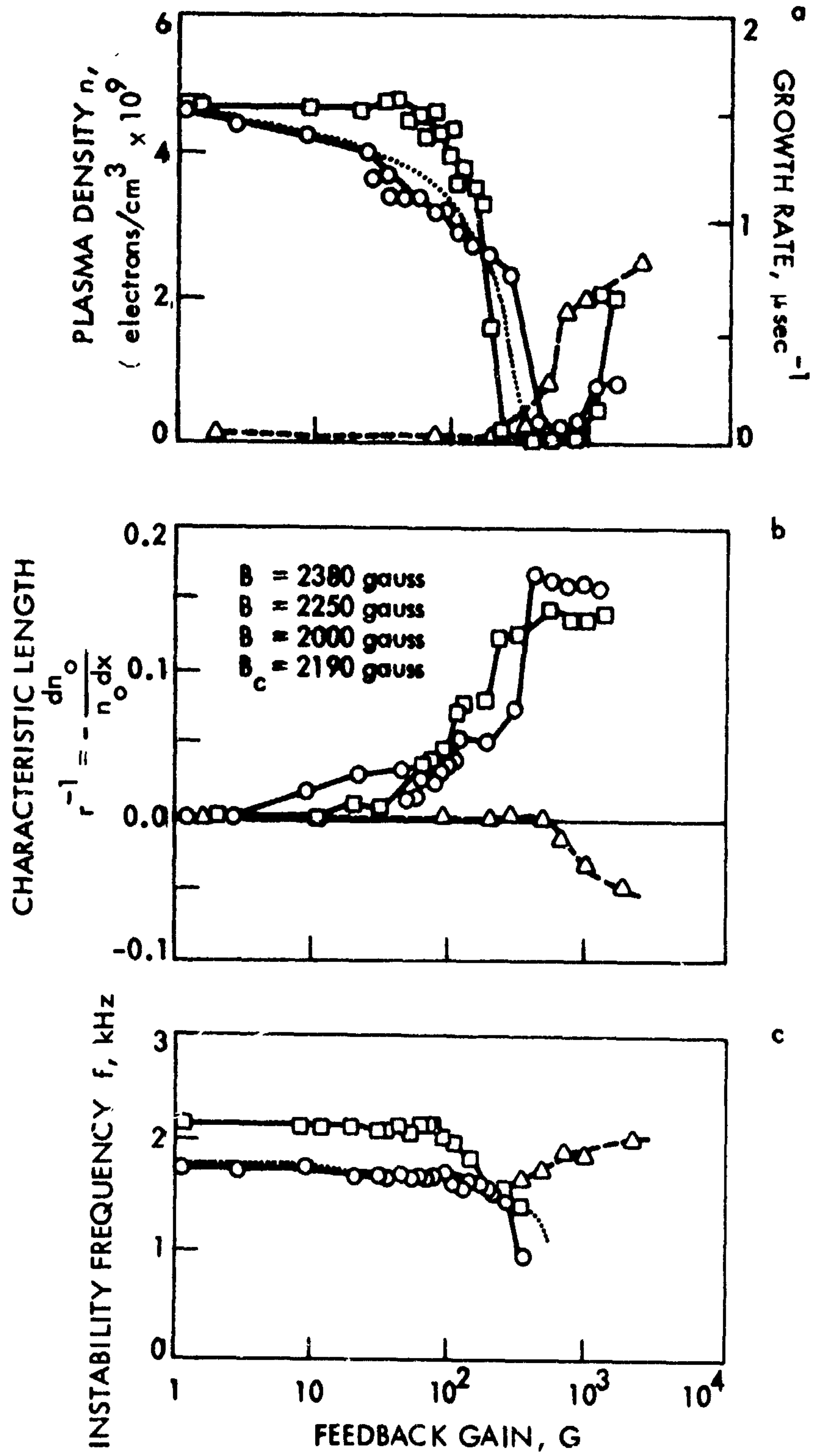

Fig. 8. Feedback gain vs: (a) plasma density; (b) characteristic length of the density gradient; and (c) instability frequency.

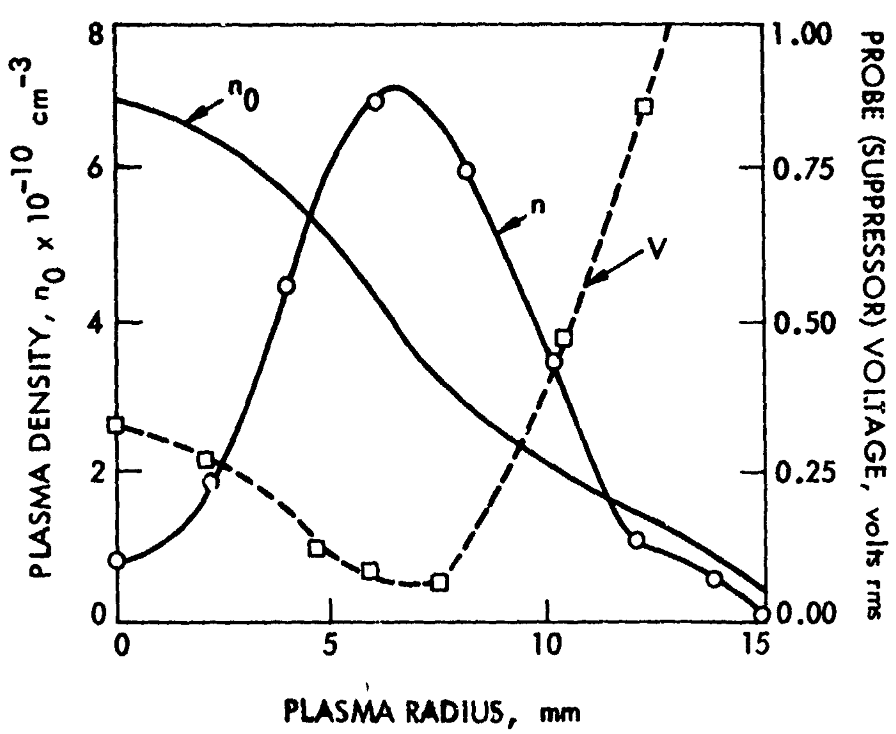

Fig. 9. Radial distribution of plasma density, n_0, instability amplitude (density), n, and required minimum probe (suppressor, rms), voltage, V, for feedback stabilization.

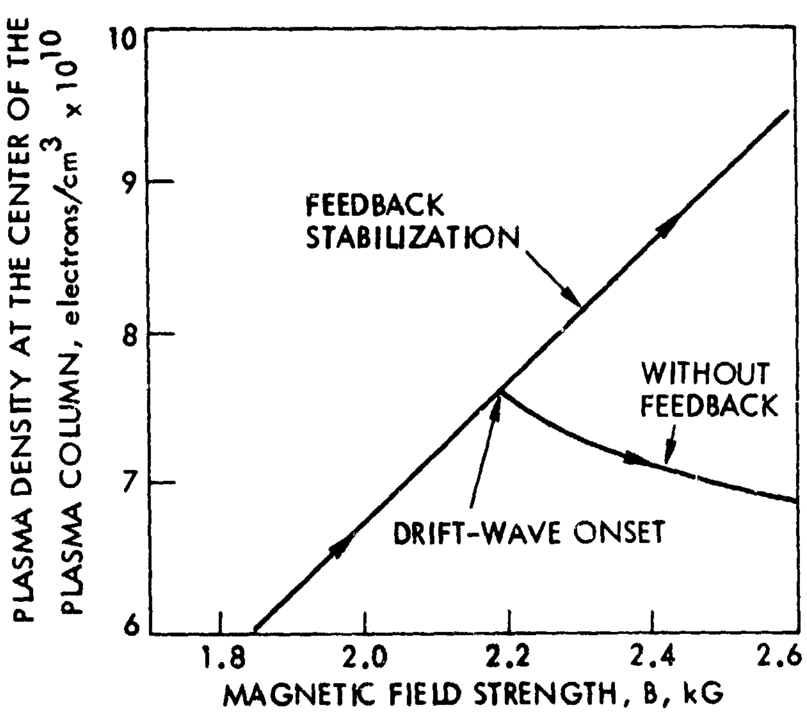

Fig. 10. Plasma density at the center of the plasma column as a function of magnetic field B with and without feedback stabilization of collisional drift waves.

of the plasma. For a thermonuclear plasma, the injection of a neutral atom beam to provide feedback-controlled volume sources of particles and momentum density is a basically different approach from the electrostatic feedback. Any perturbation imparted to the plasma by a neutral atom beam can be sensed by optical or microwave techniques.

By using the equations of motion of ions and electrons (i.e., two-fluid equations for the plasma), and the equation of continuity with source terms of density and momentum, after linearization and simplification, a quadratic equation of the dispersion relation for frequency ω is given by[61]

$$\omega(\omega - \omega_i - \omega_f) + i\omega_s(\omega - \omega_e - i\omega_f + \frac{k_\perp v_x}{\omega_c}\omega_f) + \frac{v_x}{r}\omega_f = 0 \quad (4.1)$$

where

$$\omega_e = k_\perp v_{de},$$

$$\omega_i = k_\perp v_{di},$$

$$\omega_s = \left(\frac{k_z}{k_\perp}\right)^2 \frac{B}{Mn_o\eta},$$

$$\omega_c = \frac{qB}{M},$$

$$v_{di} = \frac{T_i}{qBr} = \text{ion diamagnetic drift velocity},$$

$$v_{de} = \frac{T_e}{qBr} = \text{electron diamagnetic drift velocity},$$

$$\omega_f = S/n_i = \text{feedback frequency or gain},$$

$$v_x = \text{neutral atom beam velocity},$$

$$M = \text{neutral atom mass},$$

$$r^{-1} = -\frac{dn_o}{n_o dx},$$

n_o = equilibrium plasma density,

n_1 = neutral atom density,

S = neutral atom beam source,

η = plasma resistivity related to ω_s

$T_i, T_e, q, B, k\ , k_z$, etc., have been defined.

For $\omega_s >> |\omega_e|$ and $\omega_f = 0$, Eq. (4.1) reduces to

$$\omega^2 - \omega(\omega_i - i\omega_s) - i\omega_s\omega_e = 0.$$

This has the two solutions

$$\omega_a = \omega_e + i\omega_e(\omega_e - \omega_i)/\omega_s, \text{ and } \omega_b = \omega_i - \omega_e - i\omega_s \qquad (4.2)$$

The purpose of the analysis is to use the neutral atom beam to stabilize ω_a without destabilizing ω_b.

There are three basically different stabilization mechanisms which are characterized by the phase (angle) shift θ and the relationships between v_x and $r\omega_e$ (or normalized beam momentum $v_x/r\omega_e$) of the neutral atom beam feedback system:

- Plasma density smoothing, i.e., smoothing density perturbation with the neutral-beam injected plasma. For this case $|v_x| << |r\omega_e|$ and $\theta = 180^o$. The marginal stability requirement in Eqs. (4.1) and (4.2) is $\omega_f = \omega_s(\omega - \omega_e)/\omega$. For $\omega_s >> |\omega_e| >> |\omega_f|$, this becomes $\omega_f = \omega_e(\omega_e - \omega_i)/\omega_s = -\gamma$ = the instability growth rate.
- Equivalent (or simulating) minimum B configuration $|v_x| >> |r\omega_e|$ and $\theta = 0^o$. The marginal stability condition is $\omega_f = -\omega_e(\omega_e - \omega_i)r/v_x$. The pressure exerted by the momentum of the neutral beam for this

case becomes important and produces an equivalent or simulating minimum B effect.

- De-energization of resistive drift waves. For this case $|v_x| \gg |r\omega_e|$, $\theta = -90^\circ$. The marginal stability requirement is the feedback gain $\omega_f = i\gamma\omega_c/k_\perp v_x$. The momentum of the neutral atom beam exerts pressure which reduces the energy of the waves in the plasma.

Comparison of the effectiveness of the three mechanisms indicates that feedback gain required for stabilization by mechanism (a), density smoothing, is rather large. Momentum control of the neutral beam, mechanism (b), can be more effective than density control if $v_x \simeq v_i$. At the same time, mechanism (b), as equivalent minimum B configuration may have a relatively long energy replacement time of the plasma. The alternative momentum control, mechanism (c), can be better than mechanism (b) for very small instability growth rate, γ/ω_e, but it is less effective than mechanism (a).

In a numerical example, it is assumed that $\omega_e = -\omega_i > 0$, $b = \frac{1}{2}(k_\perp a_i)^2 = k_\perp^2 \frac{T_i}{M\omega_c^2} = 2(10^{-2})$ or 10^{-6}, $\theta = \arg(\omega_f/\omega_e)$, $\omega_s/\omega_e = 20$ or 200, $v_x/r\omega_e = 650$ - normalize beam momentum. By introducing these values into Eqs. (4.1) and (4.2), the calculated values of instability growth rate and minimum feedback gain required for stabilization are obtained. Figure 11 shows the growth rate varying with phase shift for various values of the normalized feedback gain ω_f/ω_e. Figure 12 indicates the minimum feedback gain $|\omega_f/\omega_e|_{min}$ required for stabilization varying with the normalized neutral beam momentum $v_x/r\omega_e$. It is seen that the value of $|\omega_f/\omega_e|_{min}$ decreases as ω_s/ω_e increases when the feedback stabilization for resistive drift wave is concerned.

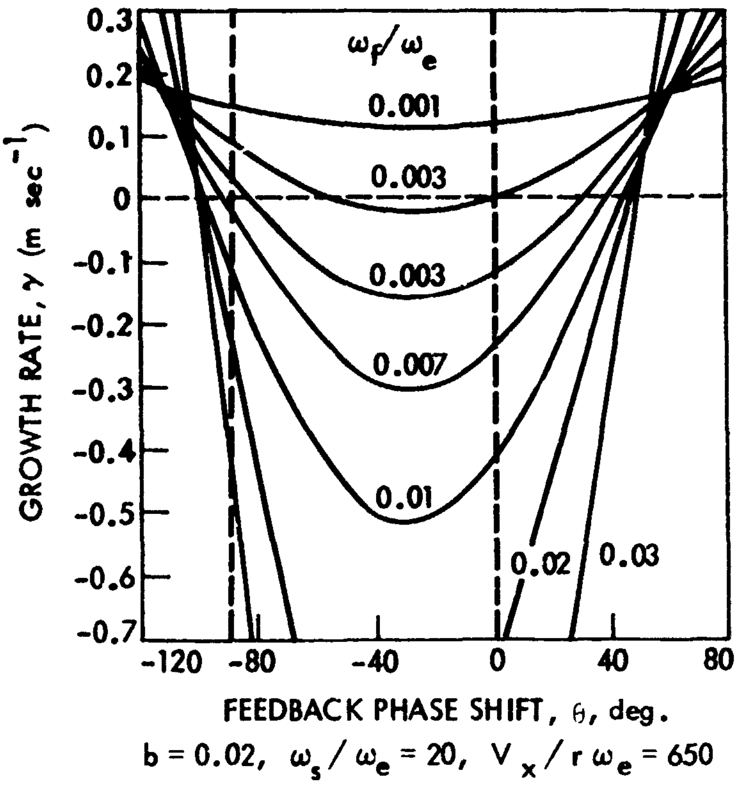

Fig. 11. Growth rate vs phase shift for different values of normalized feedback gain, ω_f/ω_e.

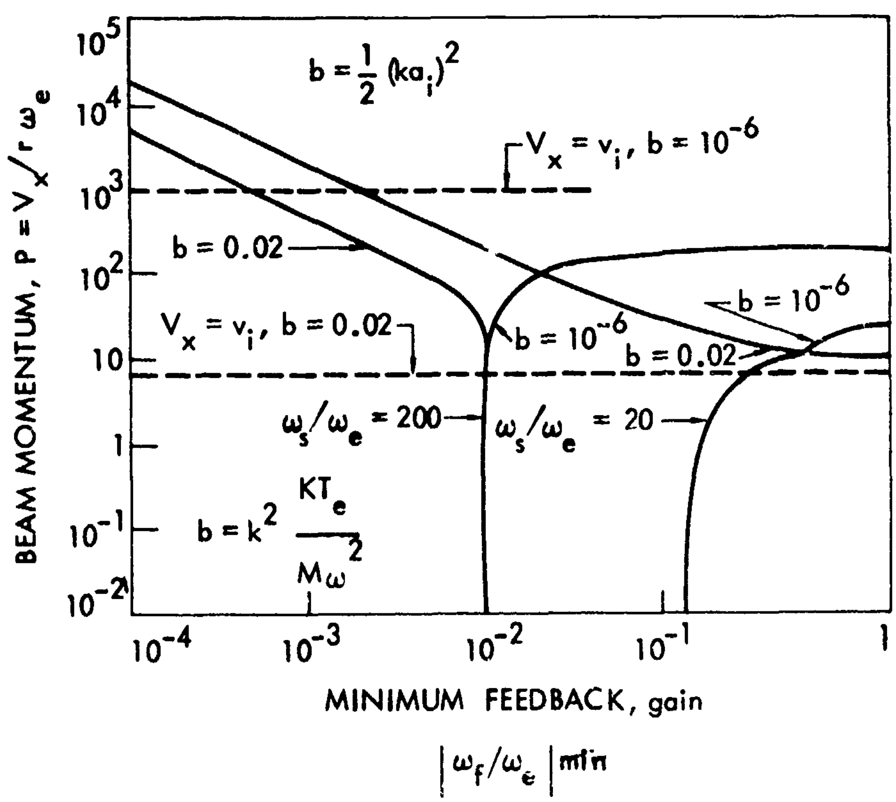

Fig. 12. Minimum feedback gain required for stabilization vs normalized neutral beam momentum.

4.3 Modulated Microwave Energy Sources

The possibility of feedback stabilization for resistive drift waves of a thermonuclear plasma by the remote injection of neutral atom beams has been analyzed above. In this section some experiments to demonstrate remote feedback control of collisional drift waves by modulated microwave energy sources is discussed. The experiment is performed in a cesium plasma of the Q-machine with $T_e = 0.24$ eV, $n_e = 5(10^{10})$ cm^{-3} and B = 4.0 kG. A half-wave dipole antenna (located about 2 cm outside the plasma column) is used as a remote suppressor, and a Langmuir probe is used as a detector. The amplified and phase-shifted ion-saturation current instability signal is introduced to modulate the microwave energy source so that the microwave power output is proportional to the instability amplitude. The microwave energy heats the plasma by resonance absorption.[66]

For optimum stabilization, the modulated microwave power must be applied to the region of maximum instability amplitude and approximately 180° out of phase with the local density oscillation. The measured results from the plasma for (a) the dependence of instability amplitude and frequency on feedback phase shift and gain, (b) the radial distribution of plasma density, and (c) the change of magnetic field strength on plasma stabilization with microwave feedback are similar to those obtained by the electrostatic feedback (see Figs. 7-10).

4.4 Modulated Uniformly-Distributed Electron Sources

An inhomogeneous plasma confined by a main magnetic field B_z in the axial direction is considered. The particle density n varies in the x-direction. The feedback stabilization with modulated electron sources uniformly distributed in the plasma is applied to a very weakly collisional regime where the electron mean free path is greater than the parallel wavelength. In this case the motion of electrons can be described by the kinetic equation in the drift approximation. The kinetic equation for perturbed electron distributions with collision and source terms in the simplest form is given by

$$\frac{\partial f_{ie}}{\partial t} + \frac{E_y}{B_z}\frac{\partial f_{oe}}{\partial x} - \frac{qE_y}{m}\frac{\partial f_{oe}}{\partial v_z} + v_z\frac{\partial f_{ie}}{\partial z} = -\nu_e(f_{ie} - n_{ie}f_M) + S_e \tag{4.3}$$

where

$f = f(\vec{r}, \vec{v}, t)$ = the Boltzmann distribution function (see section 2.4)

f_{oe}, f_{ie} = the zero-order and first-order distribution functions

E_y = electric field in the y-direction

m, q = the electron mass and charge

n_{oe}, n_{ie}, v_e, ν_e = the electron densities, velocity, and collision frequency

$f_M = f_{oe}/n_{oe}$ = the Maxwelliam velocity distribution.

The feedback electron source, S_e, is distributed throughout the plasma and responds linearly to the local electron density perturbation. Based on this assumption, the source term can be given by[59,60,69]

$$S_e = -\omega_f n_{ie} f_M \tag{4.4}$$

By the Fourier component analysis of Eq. (4.3) with the aid of Eq. (4.4), solving for f_{ie} and integrating, the ratio of the electron density at local perturbed state to the electron density at initial equilibrium state, n_{ie}/n_{oe}, is obtained

$$\frac{n_{ie}}{n_{oe}} = \frac{q\phi}{T}\,\frac{1 - (\omega - \omega^* - i\nu_e)\int\left[f_M dv_z/(\omega - k_z v_z + i\nu_e)\right]}{1 - i(\omega_f - \nu_e)\int\left\{f_M dv_z/(\omega - k_z v_z + i\nu_e)\right\}} \qquad (4.5)$$

in which

$q\phi = q\phi$ = work function, ϕ = scalar potential,

T = kinetic temperature,

$$\omega^* = -\frac{k_y T}{qB_z}\,\frac{dn}{ndx},$$

k_y, k_z, ω = have been defined previously.

In the case of short mean free path of electrons (for relatively strong collisions), $\omega \ll k_z v_{Te} \ll \nu_e$ and $\omega\nu_e \sim (k_z v_{Te})^2$, Eq. (4.5) reduces to

$$\frac{n_{ie}}{n_{oe}} = \frac{q\phi}{T}\,\frac{\omega^* + i(k_z v_{Te})^2/\nu_{ei}}{\omega + i\omega_i + i(k_z v_{Te})^2/\nu_{ei}} \qquad (4.6)$$

where $v_{Te} = (2\ T/m)^{\frac{1}{2}}$, electron thermal velocity, and ν_{ei} = electron-ion collision frequency. In the case of long mean free path of electrons (for relatively weak collisions), $\nu_e \ll k_z v_{Te}$ and $\omega \ll k_z v_{Te}$, Eq. (4.5) becomes

$$\frac{n_{ie}}{n_{oi}} = \frac{q\phi}{T}\,\frac{1 + i(\pi/2)^{\frac{1}{2}}\ (\omega - \omega^* + i\nu_e)/(|k_z| v_{Te})}{1 + (\pi/2)^{\frac{1}{2}}\ (\omega_f - \nu_e)/(|k_z| v_{Te})} \qquad (4.7)$$

If the ion density is also perturbed by the electron sources, the ratio of ion density at local perturbed state to ion density at initial equilibrium state, n_{li}/n_{oi}, is

$$\frac{n_{li}}{n_{oi}} = \frac{q\phi}{T}\,\frac{\omega^* - b\omega}{\omega + b\omega} \qquad (4.8)$$

where $b = \frac{1}{2} k^2 a_i^2$ as defined previously.

For $b \simeq \omega_f / k_z v_{Te} \ll 1$, from the quasineutrality condition the dispersion relation is obtained.

$$\omega = \omega^* \left[\left\{ 1 - 2b + \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{\omega_f}{|k_z| v_{Te}} \right\} + i\left(\frac{\pi}{2}\right) \frac{\omega^*}{|k_z| v_{Te}} \left\{ 2b - \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{\omega_f}{|k_z| v_{Te}} \right\} \right] \tag{4.9}$$

Feedback stabilization can be achieved with purely imaginary feedback frequency ω_f such that $\arg(\omega_f/\omega^*) = - 90^{\circ}$ (or $+ 270^{\circ}$), and gain required for

$$|\omega_f \omega^*| > 2b(\omega^*)^2 \tag{4.10}$$

Feedback stabilization can also be achieved with phase shift 0° if the condition

$$\left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{\omega_f}{|k_z| v_{Te}} > 2b$$

is satisfied. Thereofore, the optimum phase shift for the feedback stabilization will be between $- 90^{\circ}$ (or $+ 270^{\circ}$) and 0°. The electrostatic feedback by phase shift of $- 90^{\circ}$ and the neutral atom beam feedback by phase shift of 0° are corresponding to the phase shift of the modulated electron source feedback. Although the approaches of the feedback stabilization are different, in general, the feedback stabilization by the modulated electron sources is in good agreement theoretically and experimentally with the corresponding stabilizing effects by the electrostatic, neutral atom beam momentum or modulated microwave energy source feedback.

5. MAIN COMPARISONS AND CONCLUSIONS

5.1 Main Comparisons

In the magnetic field configuration stabilization, the internal axial magnetic field, the rotational transform, and the external axial magnetic field of screw (bumpy or diffuse) pinch together with the azimuthal magnetic field to form some fundamental elements of magnetic fields for plasma stabilization. The magnetic shear and the magnetic well (including minimum B for open-end system and minimum average B for closed-end system) have shown good results to control the hydromagnetic (MHD) and drift instabilities. The multipole configuration consists of the linear and toroidal multipoles. The linear multipole device has been operated as a quadrupole in the collisionless regime. The toroidal multipoles have been studied from low β equilibrium and flute instability to high β-ballooning stability. The magnetic shear, the magnetic well, and the multipoles can be considered as the advanced magnetic field configuration for plasma stabilization.

The high magnetic power, P_{mag}, required per unit length of the confinement chamber for these configuration is approximately given by

$$P_{mag} \simeq C \frac{B^2 R \eta}{NA} \qquad (5.1)$$

where

B = the magnetic induction, or magnetic field strength,

R = the coil radius,

A = the cross-sectional area of the coil,

N = the number of coil turns per unit length of the confinement chamber,

η = the resistivity of the coils, and

C = a constant.

Among the dynamic stabilization methods all use oscillating electric field or alternating current, except the electron temperature modulation method, to stabilize the drift waves of a thermonuclear plasma. In the oscillating electromagnetic field method and the oscillating non-uniform electric field method, both electric and magnetic fields are considered. In the electron temperature modulation method, microwave (or radiofrequency field) and optical energy sources can be employed to heat the plasma by remote control. The oscillating magnetic field for use in the dynamic stabilization is usually produced by alternating current or currents.

Of the methods discussed above, the dynamic stabilization for drift instability of a collisionless, inhomogeneous plasma is most effective. The axial alternating current method used for dynamic stabilization, however, is potentially effective for collision-dominated drift waves of the plasma.

Dynamic stabilization has demonstrated the possibility of stabilizing low-frequency drift waves (or broad turbulences) and improving containment time of a thermonuclear plasma. Apart from electron temperature modulation by microwave heating, modulated neutral beam or optical energy source (such as weak laser beam) may be useful for dynamic stabilization of a fusion reactor by the remote control and detection methods.

The power required for dynamic stabilization, P_{dyn}, by the high-frequency electric field at frequency Ω is given by[70]

$$P_{dyn} = bn_o T\Omega \ . \qquad (5.2)$$

where

n_o = equilibrium plasma density,

T, b = as defined previously, and

bn_oT = a fraction of the internal (or thermal) energy of the plasma.

Electrostatic feedback for which feedback elements immersed in, or in contact with, plasma have proved effective to stabilize plasma instabilities and improve plasma confinement. Particularly, the collisional drift instability in the oscillatory regime has been stabilized by immersing electrostatic probes to draw electric current from the plasma. Since material probes cannot resist high temperature in a thermonuclear plasma, the electrostatic feedback method is probably limited to laboratory plasmas of moderate density and temperature.

The application of feedback control to a nuclear fusion reactor, however, depends critically on the possibility of remote control and detection method. Therefore, the remote control of neutral atom beam feedback to stabilize resistive drift waves and the modulated microwave energy sources to stabilize collisional drift waves has been proposed. Charged particle beams and optical beams (including laser beam) can also be used for remote control and detection in the feedback stabilization processes.

The effectiveness of some experimental results on the low-frequency drift waves and the good qualitative agreement with theoretical expectation indicate that the study of feedback stabilization (as well as dynamic stabilization) may be a fruitful approach to increase the containment time of a thermonuclear plasma.

The power required for linear feedback stabilization, P_{fb}, by the parallel electron current is given by

$$P_{fb} = a^2 bn_o T\omega_f \tag{5.3}$$

in which $\omega_f = \omega_{in}$ = instability frequency, $a = q\phi/T \simeq n/n_o$ = normalized instability amplitude, and $bn_o T$ remains a fraction of the internal (or thermal) energy of the plasma.

The ratio of power required for the feedback stabilization to that required for the dynamic stabilization on the basis of Eqs. (5.2) and (5.3) is

$$P_{fb}/P_{dyn} = a^2\omega_f/\Omega, \tag{5.4}$$

which is much smaller than unity when $a \ll 1$ and $\omega_f < \Omega$ in the most practical cases.

The high magnetic power requirement of the advanced magnetic field stabilization system is equivalent to the total magnet loss per unit length of the confinement (or vacuum) chamber due to heating, confinement, and stabilization of the plasma.

For purposes of comparison and conclusions, the main advantages and disadvantages of the plasma stabilization systems are given below:

5.1a Magnetic Field Configuration Stabilization

- Provision for plasma heating, confinement and stabilization
- Self-stabilization in appropriate magnetic field configuration
- Suppression of the hydromagnetic (or MHD) instabilities in particular

- Complex stabilizing magnetic fields required
- Complex winding and high cost of the solenoidal conductor
- High power required for magnetic field generation and operation

5.1b Dynamic Stabilization

- Possibility of stabilizing many modes, different low-frequency drift instabilities, and broad-based turbulences simultaneously
- Suppression of low-frequency drift instabilites in particular
- Stabilizing frequency much higher than instability frequency and no phase shift needed
- Excessive power required to maintain new dynamically stable equilibrium for high growth-rate instabilities.

5.1c Feedback Stabilization

- Relatively simple magnetic geometries required when appropriate feedback and phase shift systems are selected.
- Lower power requirement for stabilization because when the amplitude of instability is reduced, the feedback amplitude reduces accordingly.
- Feedback can interact only with one unstable mode near marginal instability; it is difficult to stabilize many modes simultaneously.
- Application of feedback control by means of material probes and electrodes limited only to plasmas of moderate density and temperature. Remote control and detection methods are preferred.
- Linear feedback (feedback frequency $\omega_f = \omega_{in}$ instability frequency with specific phase angle) can stabilize different drift instabilities and the $m = 1$ flute instability.[67-68]

- Nonlinear feedback (repetition rate $\leq$ instability frequency, with specific phase angle) effective to suppress perturbations in the nonlinear feedback constant corrective signal is applied whenever the instability amplitude grows above a prescribed level. Higher modes, however, are easily destabilized.

5.2 Conclusions

- In the magnetic field configurations, the principles of magnetic shear and magnetic well will continue to play an important role in plasma stabilization. The trend of the magnetic shear and the magnetic well is toward more sophisticated and more precisely controlled economic magnetic field geometry to stabilize hydromagnetic and drift instabilities.
- Dynamic stabilization ($\Omega \gg \omega_{in}$, no phase requirement) can stabilize many modes, different low-frequency drift instabilities, and broad-band turbulences simultaneously. Excessive power, however, is required to maintain new dynamically stable equilibrium for high growth-rate instabilities.
- Feedback stabilization (selected frequency and specific phase shift required) can interact only with a part of the plasma and one unstable mode near marginal instability. Feedback control by means of material probes and electrodes is limited to plasmas of moderate density and temperature. Linear feedback ($\omega_f = \omega_{in}$) can reduce drift instabilities effectively and suppress the $m = 1$ mode of flute instability reasonably. Nonlinear

feedback (repetition rate $\leq$ instability frequency ω_{in}) is effective to suppress low-frequency drift instabilty at repetition rates below instability frequency but higher modes are easily destabilized.

- There are more basic differences between dynamic stabilization and feedback stabilization. In the dynamic stabilization the high frequency (or radio-frequency) field interacts with the entire plasma at a new equilibrium density n_o while in the feedback stabilization the plasma equilibrium remains unchanged and the time-varying suppression signal interacts only with the unstable part of the plasma. Both dynamic stabilization and feedback stabilization, however, will prefer to have remote control and detection methods for use in a controlled fusion reactor, even a rather advanced and costly technique is required for remote dynamic or feedback control.
- At present, plasma self-stabilization in appropriate economic magnetic field configurations is still uncertain. If the plasma self-stabilization can prove to be fully effective in the laboratory scale, the actual design of a controlled fusion reactor with the provision of complex stabilizing magnetic fields will require some reliable scaling laws to determine the reactor size. The reliable scaling laws for controlled fusion reactor design, however, have not satisfactorily been found. Since the magnetic field configuration stabilization is mostly effective in stabilizing the hydromagnetic instability. The dynamic or feedback stabilization which can stabilize the drift (gross or universal) instability of a thermonuclear plasma may contribute to controlled fusion research.

- It is expected that the research and development of remote control and detection method for potential dynamic stabilization or feedback stabilization of a thermonuclear plasma will probably become important in developing a controlled fusion reactor. Either the dynamic stabilization or the feedback stabilization may be combined with the magnetic field configuration to stabilize a thermonuclear plasma so that the minimum requirement for a controlled fusion reactor may be attained and satisfied.

In the preceding discussion and analysis a number of new ideas which can be developed into original methods for understanding plasma instability and improving plasma stabilization have been introduced. The main fields of the new ideas may consist of the combination of magnetic shear and magnetic well, and the combination of magnetic field configuration and dynamic or feedback stabilization with remote control and detection methods. In the main fields, some specific ideas such as the combined effect of rotational transform, magnetic shear and minimum $\bar{B}$ on a toroidal plasma, extention of the present magnetic shear analysis, dynamic stabilization by remote microwave (or radiofrequency) field control (see Fig. 6), etc., can be developed to make contributions for a controlled fusion reactor.

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